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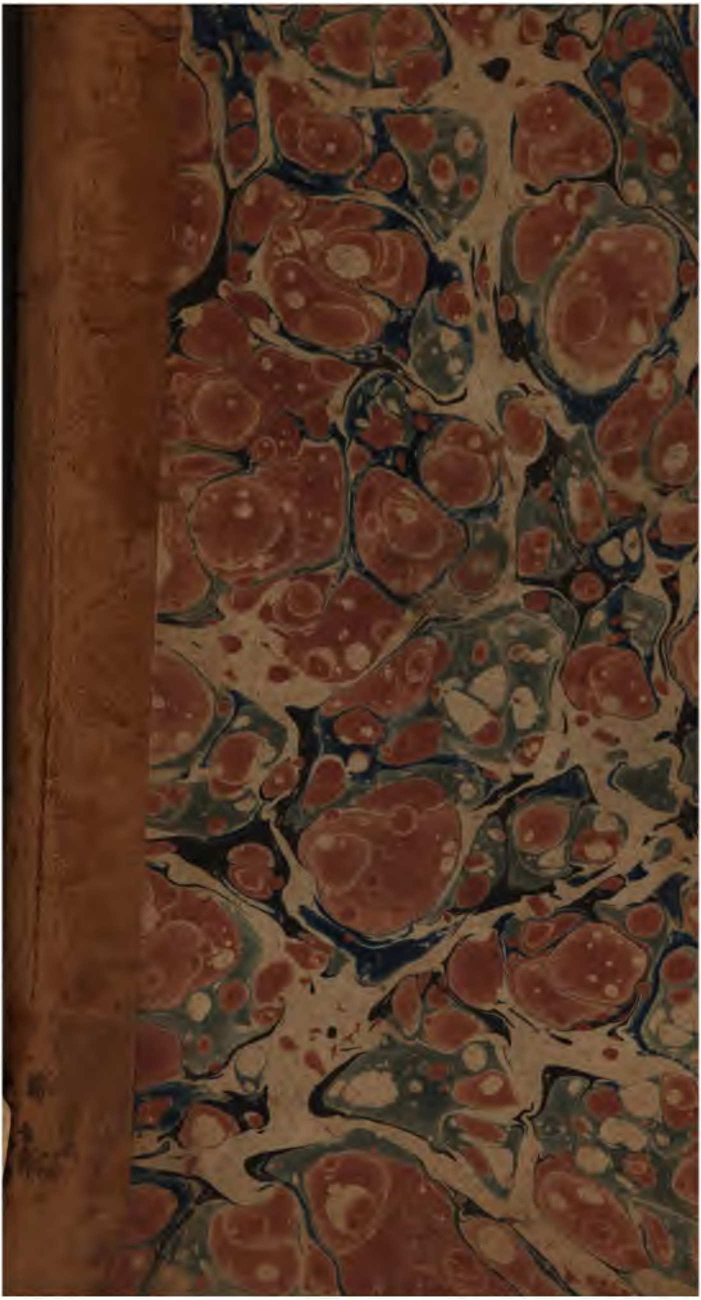

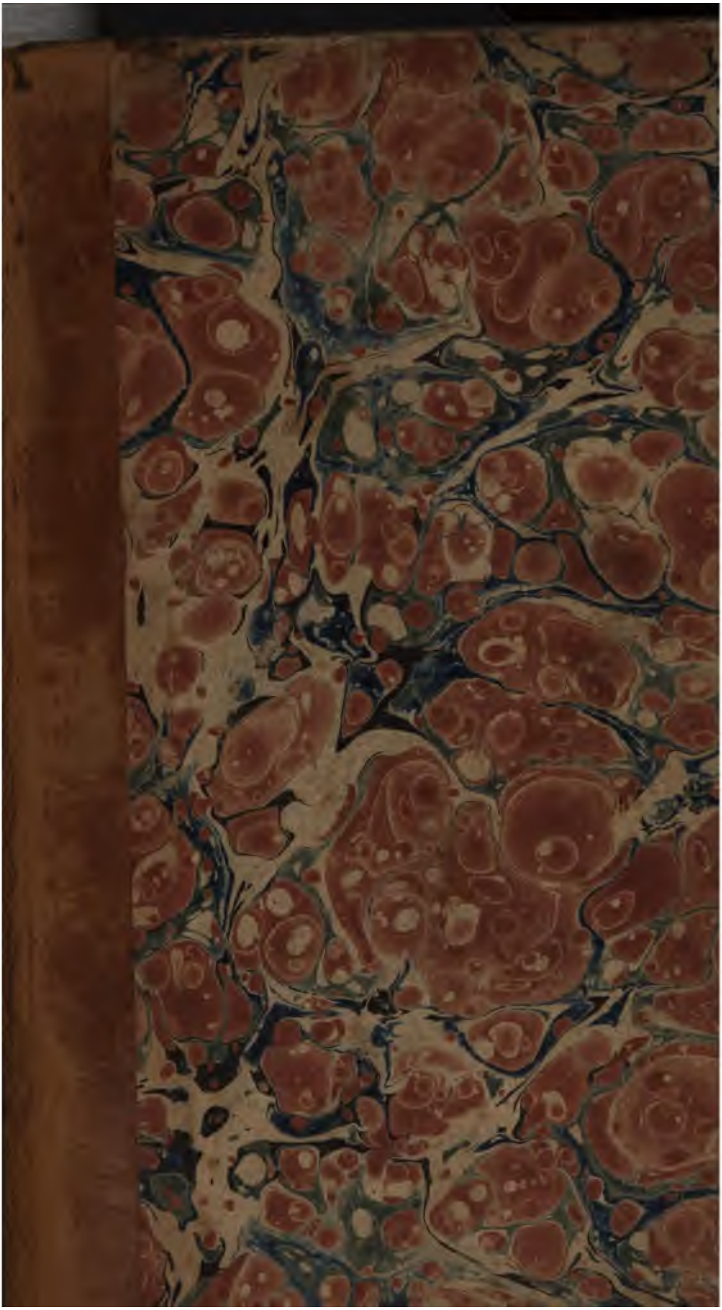

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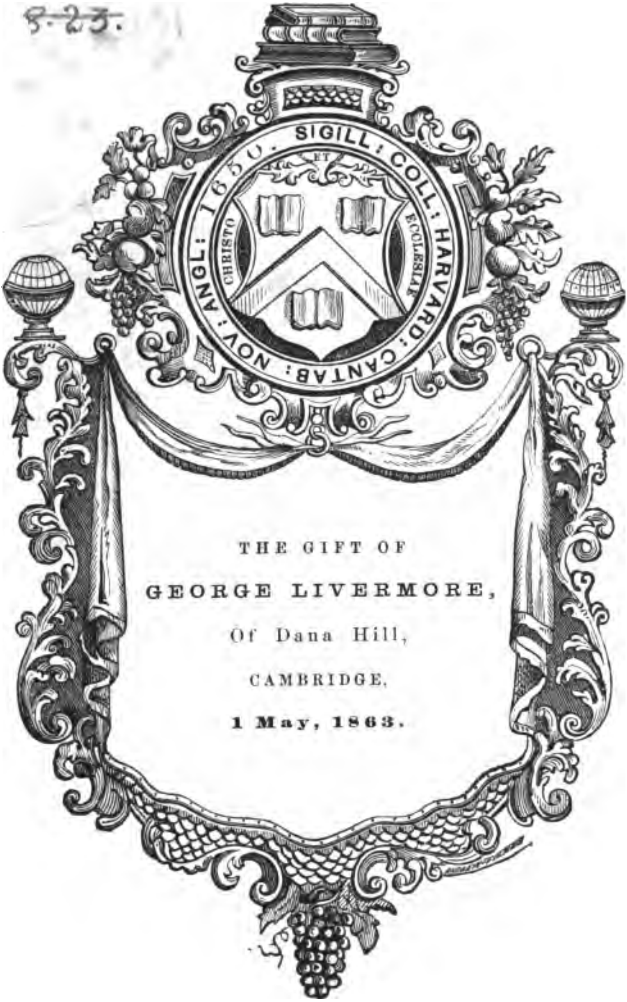
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No. 14

# CONVERSATIONS

P. Mackintosh Jr.

ON

# ARITHMETIC,

WITH

*Demonstrations to each Rule,*

IN EASY AND FAMILIAR LANGUAGE;

THE WHOLE OF WHICH IS DESIGNED TO RENDER THE

STUDY OF ARITHMETIC

PLEASING AND INSTRUCTIVE TO THE PUPIL.

BY LEONARD PEIRCE.

BOSTON:
PRINTED BY LINCOLN & EDMANDS, FOR THE AUTHOR.
Sold by them, and at the several Bookstores.
1823.

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1863, May 1.
Gift of
George Livermore,
of
Cambridge.

DISTRICT OF MASSACHUSETTS, *to wit:*

*District Clerk's Office.*

**BE IT REMEMBERED,** That on the twelfth day of November, A. D. 1823, and in the forty-eighth year of the Independence of the United States of America, LEONARD PEIRCE, of the said district, has deposited in this office the title of a book, the right whereof he claims as Author, in the words following, to wit:

"Conversations on Arithmetic, with Demonstrations to each Rule, in Easy and Familiar Language; the whole of which, is designed to render the Study of Arithmetic pleasing and instructive to the Pupil. By LEONARD PEIRCE."

In conformity to the Act of the Congress of the United States, entitled, "An Act for the Encouragement of Learning, by securing the Copies of Maps, Charts and Books, to the Authors and Proprietors of such Copies, during the time therein mentioned;" and also to an Act entitled, "An Act supplementary to an Act, entitled, An Act for the Encouragement of Learning, by securing the Copies of Maps, Charts, and Books, to the authors, and Proprietors of such Copies during the times therein mentioned; and extending the benefits thereof to the Arts of Designing, Engraving and Etching Historical, and other prints.

JOHN W. DAVIS,
*Clerk of the District of Massachusetts.*

# PREFACE.

Some apology will perhaps be required for publishing a work of this kind, when so many arithmetics are already before the public. The only apology which I have to offer, is a desire to render the study of arithmetic more beneficial to scholars. It has become almost a universal practice, particularly in the *country*, for scholars to pass through their arithmetic, without obtaining any thing more than a practical knowledge of the rules. This is owing, in a great measure, to the multiplicity of studies, which have, of late, been introduced into schools. When our present system of schooling was established, and indeed, till within a few years, the only branches taught, were reading, writing, arithmetic, and English grammar. The number of scholars was generally less than at present, and instructers were then enabled to proceed more methodically, and to bestow more attention on each branch, than the present state of schools, in general, will admit of. In most schools at present, the usual studies, in addition to those before studied, are geography, rhetoric, logic, history, and frequently several others, which, with the increase of scholars, compel an instructer to do every thing in a hurry, and consequently, to do nothing well. These considerations led me to believe, that an arithmetic on the plan of the following work, would be of public utility. By it, scholars will be able to obtain a knowledge of the first principles of arithmetic, with but very little assistance from an instructer. By this, is not meant, that a thorough knowledge of the theory of arithmetic may be obtained by a mere perusal of the demonstrations; this is not practicable. In order for this, the demonstrations must be attended to closely, and for a considerable time. It should be no discouragement to a scholar, that he cannot at first understand the demonstrations. If he begins with a determination to understand them, and obtains a thorough knowledge of the simple rules, he will have little difficulty in the succeeding rules. In all arithmetics adapted to the use of schools and academies, which I have seen, the demonstrations are given algebraically, or there are no demonstrations at all. In the former case, scholars consider the demonstrations too difficult for them to understand, and in the latter, that nothing more is required of them, than is contained in the book. The authors whom I have consulted, in making this compilation, are, Pike, Webber, Lacroix, and Adams. In many of the rules, particularly Fractions, I have derived more benefit from Lacroix than from the others. Pike and Webber, I have found of much advantage in demonstrating those rules which are not contained in Lacroix. Adams's demonstration of extracting the Square and Cube Roots, I have found to be superior to any which I have seen in any former author. From all these authors, and from Adams more than either of the others, I have extracted examples when they would answer my purpose. I have calculated the work entirely in the Federal Currency, because I conceived it to be the only one which is generally, or which ought to be taught in our schools. The questions in each Conversation, are designed for the examination of the pupil by the instructer; and a scholar should never be allowed to leave a Conversation until he can answer all these questions correctly. The method of stating the Single and Double Rules of Three, is different from the usual method; but I conceive it to be more simple, and more agreeable to the nature of Proportion than the ordinary way. In this I have followed Lacroix. I have also followed the same author, in the multiplication of Duodecimals, and consider it more analogous to multiplication in general, than the common method. Errors in a work like this must be expected; the press cannot entirely avoid them; and several, it is very probable, have escaped my observation. These, it is hoped, the candid will forgive; from whom, any intimations of errors or amendments will be thankfully received by

*Millbury, (Mass.) Oct.* 1823. THE AUTHOR.

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# CONVERSATIONS
# ON
# ARITHMETIC.

## CONVERSATION I.

### NUMERATION, SIMPLE ADDITION, AND SIMPLE SUBTRACTION.

TUTOR AND PUPIL.

*Tutor.* What is Arithmetic?

*Pupil.* Arithmetic, I think, is about numbers, and shows how to work with them so as to obtain correct answers to arithmetical questions.

*Tut.* You are right. Arithmetic is the art or science which teaches how to compute by numbers. Without a knowledge of it, you will not be able to transact business correctly in any calling of life, and you will always be liable to be defrauded by those who are better informed. This ought to excite you to make great exertions to become familiar with every part of Arithmetic, necessary for a man of business. At first it may seem a hard task, but, with a little patience, in learning the fundamental rules, you will find arithmetic to be a very interesting study.

*Pup.* I am always pleased to receive instruction from you, and whatever instruction you may give me, I will not let pass without my utmost endeavours to understand.

*Tut.* Can you tell of how many kinds arithmetic is?

*Pup.* Arithmetic is of two kinds, theoretical and practical.

*Tut.* Very well. And can you tell how these kinds differ?

*Pup.* The theory of arithmetic explains the nature and quality of numbers, and shows the reasons of practical operations. The practical part shows the most useful and expeditious method of working by numbers.

*Tut.* What is Numeration?

*Pup.* Numeration, I think, shows how to read numbers.

*Tut.* You have a right idea of it, but you ought to have a thorough knowledge of it before you proceed further.

1. Numeration is that part of arithmetic which teaches how to write any given numbers, by means of certain characters, and how to read those characters when written. Numbers are expressed by the following characters.

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| one, | two, | three, | four, | five, | six, | seven, | eight, | nine. |

2. When a number greater than nine is to be expressed, it is done by tens, and these tens are expressed by one of the above characters, having another character placed at its right. If it be required to write twelve, it is done by writing the figure 1, and at its right, write the figure 2, thus, 12. The one at the left hand being removed towards the left, one place, is in the place of tens, and expresses ten units, or ten times as much as 1 expresses when it stands alone; the 2 at the right expresses only two, it being in the place of units. In this manner twelve is expressed. When it is required to write eleven, you write two 1s thus, 11; the one at the left expressing ten, and the 1 at the right expressing one. If it be required to write one hundred and eleven, you write three 1s, thus 111; the 1 at the right expresses one as in the former case, and likewise the 1 in the middle expresses ten; but the 1 at the left expresses one hundred, it being removed one place further to the left than the middle figure, which is in the place of tens. The number twenty-two is written 22; the 2 at the left expresses twenty, and the 2 at the right expresses two. The expression two hundred and twenty-two is written 222, and all the figures increase in the same manner; every removal to the left increasing the value of the figure *ten times.*

"This is the fundamental law of our written numeration, that *a removal of one place towards the left increases the value of the figure ten times.*"

3. When we wish to write any number of tens, there are none of the above characters with which we can do it; if, for instance, we wish to write ten, or twenty; we cannot, by any of the above characters, do it. If we write 1, with any of the characters at its right, they will make more than ten; and the same of the other figures. To remedy this we have a character which signifies nothing, and is called a cipher, or nought; this is the character, 0.

If we wish to write ten, we first write the figure 1, and at the right of it, write the cipher; thus, 10 expresses ten, the 1 being removed to the left, by this character which signifies nothing. If it be required to write one hundred, we must write 1, with two ciphers at its right, thus, 100; to write one thousand, three ciphers must be written, 1000; to write twenty, 2 with one cipher at its right, and so on with all the figures; any number of tens, hundreds, thousands, &c. being expressed, by first writing the significant figure, and then at its right place the proper number of ciphers.

*Pup.* I do not understand you when you say significant figure. I should like to have you explain it.

*Tut.* By significant figures are meant the figures which express some number of themselves, or all the figures except the cipher.

When we wish to write any number between one hundred and one hundred and ten, care must be taken that we place the figures according to their values. If it be required to write one hundred and one, we write it thus, 101; to write one hundred and two, thus, 102; and so on, the cipher in the middle carrying the significant figure at the left into the place of hundreds. Ciphers are frequently placed in the middle, and all other places except at the left of numbers; thus in writing any number in figures we should take care that we do not give a wrong expression by omitting the ciphers.

In general when a number is dictated, it is written by beginning at the left, and placing, one after another, the *figures according* to their values.

When a number is written in figures, in expressing it in language, it is necessary to substitute for each figure the words which it represents, and then mention the value of its place according to the following table.

| 3 | 4, | 6 | 8 | 5, | 6 | 7 | 4, | 3 | 2 | 1, | 0 | 1 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Tens of trillions. | TRILLIONS, | Hundreds of billions, | Tens of billions, | BILLIONS, | Hundreds of millions, | Tens of millions, | MILLIONS, | Hundreds of thousands, | Tens of thousands, | THOUSANDS, | Hundreds, | Tens, | UNITS, |

The figures here you see are divided by commas, into periods of three figures each, which enables us more readily to enumerate them. This division is performed by beginning at the right and placing a comma at the right of every fourth figure; sometimes the left hand figures do not amount to a full period. In the above table the left hand period consists of only two figures, and the whole table is read as follows;—thirty-four trillions, six hundred and eighty-five billions, six hundred and seventy-four millions, three hundred and twenty one thousands, and nineteen. You may express the following numbers in figures.

| | |
|---|---|
| Twelve, - - - - - - | 12 |
| Twenty, - - - - - - | 20 |
| One hundred, - - - - - | 100 |
| Five hundred and sixty-four, - - | 564 |
| Six hund. & fourteen thous. five hundred & one, | 614,501 |
| Nineteen millions and one, - - | 19,000,001 |

Express the following numbers in words.

112, 684, 5001401, 8647236, 10109824, 1800014601.

*Pup.* I think that I now understand Numeration, and shall be able to attend to Addition with success.

*Tut.* *You must* not be too positive in thinking that *you understand it,* for it is ten to one if you do not meet with numbers which you cannot enumerate. If ever you are at a loss to know how to enumerate any numbers, you must review Numeration and make every thing perfectly familiar. Before you proceed further you must examine the following characters, and make them familiar to yourself as fast as you come to the rules to which they are applied.

| | |
|---|---|
| = | The sign of equality ; as 4 and 4 added together are equal to 8, which would be expressed thus, 4+4=8. |
| + | The sign of addition ; as, 2 + 2 = 4 ; meaning that 2 added to 2 is equal to 4. |
| — | The sign of subtraction ; as, 8 — 4 = 4 ; meaning, that 4 taken from 8 leaves 4. |
| × | The sign of multiplication ; as, 3 × 3 = 9 ; meaning, that 3 multiplied by three is equal to 9. |
| ÷<br>*or*<br>) ( | The sign of division ; as, 4 ÷ 2 = 2 ; or, 2 ) 4 ( 2 ; meaning, that 4 divided by 2, gives 2 for a quotient ; or that 2 is contained in 4 twice. |
| : :: : | The sign of proportion ; as, 2 : 4 :: 8 : 16 ; meaning, that as 2 is to 4, so is 8 to 16. |

These signs are all that will be necessary in your present course of instruction.

◆

You are now prepared to be instructed in Simple Addition, and what do you think Simple Addition is?

*Pup.* Simple Addition is a rule which teaches how to put together simple numbers, so as to express them in one whole number.

*Tut.* You have formed a very correct idea of it. It is a rule by which two or more numbers of the same denomination are put together so as to be expressed in one sum.

4. In the addition of numbers the most natural way is to begin at unity, and count till you arrive at the full expression of one of those numbers, and then proceed to add a unit at a time, until you have added as many units

as the other number contains. If you were to add 8 and 4, you would begin and count, one, two, three, four, five, six, seven, eight, and ascend 4 places above 8, which would be twelve, the amount of 8 and 4. In this manner all small numbers are added together, and the amount of each addition committed to memory. But in the addition of large numbers this process is too tedious, and the endless number of sums which may be produced by addition, render it impossible to retain them in the memory. To avoid this difficulty we have recourse to the combination of units into tens, hundreds, thousands, &c. The following is the rule for

## SIMPLE ADDITION.

Place the numbers under each other so that units may stand under units, tens under tens, hundreds under hundreds, &c. and draw a line underneath.

Begin at the right hand figure of the lower number, and find the amount of all the figures of that column, and if their sum* does not exceed 9, set it down ; but if it exceeds 9, set down what the sum exceeds ten or any even number of tens ; and if it be ten or an even number of tens, write a cipher instead of the excess, and carry as many to the next figure towards the left as the sum contained tens. Proceed in this manner with every column through the whole, and at the last or left hand column, set down the whole amount of that column.

You may add together the following numbers ; 268, 346, 792, and they must be placed as follows :

$$\begin{array}{r} 268 \\ 346 \\ 792 \\ \hline 1406 \end{array}$$

*Pup.* The work is all very plain, but I do not understand why I should carry one for every ten ; I should like to have you give the reasons for carrying in this way.

---

* *The amount of* all the numbers, expressed in one whole num*ber, is called the sum* of those numbers.

*Tut.* I shall now have an opportunity to see whether you understand Numeration; for if you do not, you cannot understand the explanations which I shall give you.

According to Numeration, (3. Con. I.) you will recollect that ten units are equal to one in the place of tens; and that ten in the place of tens are equal to one in the place of hundreds, and so on, increasing towards the left in a tenfold proportion. In the above example, you first add the right hand column, or the column of units, and find their sum is 16; this means 16 units, and as ten units are equal to 1 in the place of tens, by setting down what 16 exceeds 10, and carrying 1 to the next figure, you retain the true value. If you should begin with the second column, at 9, and add up that column, which is the column of tens, you would get 19, which expresses 19 tens, or 190 units. Now if the second column contains 19 tens, and the column of units contains 16 units, and 10 units are equal to one in the place of tens, it is plain that by taking the ten which 16 contains, and adding it to the 19 tens, you express the true number of tens there are in the two right hand columns.

*Pup.* But how do we know how many tens are expressed, if the cipher will express any number of tens.

*Tut.* The cipher itself does not express any number, but the figure at the left of it. In the above example the cipher stands for the sum of that column, and the ten which was added from the column of units, which make 20; and 20 containing 10 twice, a cipher must be written, and 2 carried to the next figure; then add the left hand column, and set down the whole amount, because there are no figures at the left to which you can carry.

Again, suppose you were requested to add together four dollars and six dimes,* and six dollars and seven dimes, you would place them thus:

```
  4  6
  6  7
 -----
1 1  3
```

Here you first add together the dimes at the right, and find they amount to 13; now the figures at the left are dollars, and ten dimes at ten cents each, make one dollar, therefore, if instead of setting down the whole number of dimes, you take as many of them as it takes to make a dollar, and call it one dollar, and all there are over this number call dimes, you get your sum in dollars and dimes, expressing the same value as if you called them all dimes ; and as you have dollars to add, you can add this one dollar, to them, and express their sum in one number. In this example you have 10 dollars given, and 13 dimes, and by taking the one dollar from the dimes, and adding it to the dollars, you express all the dollars possible, and all the dimes there are over.

---

* A dime is a ten cent piece.

*Pup*. From what you have said, it appears very plain to me that carrying for every ten gives a true sum of all the numbers added, and that it would not do to carry for any other number. I should like now to have some examples, that I may make what you have said familiar.

*Tut.* I will give you some examples, and you must perform them with care, and endeavour to understand every part, as you go along, for your progress in the *succeeding* part depends greatly on understanding well the *preceding* part of arithmetic.

Add 84371, 6250, 10, 3842, and 631 together.
Answer, 95104.

Add 3004, 523, 8710, 6345, and 784 together.
Ans. 19366.

Add 66947, 46742 and 132684 together.* Ans. 246373.

◆

I will now instruct you in the next part of this conversation. And can you tell what Simple Subtraction is ?

*Pup*. Simple Subtraction, I think must relate to taking one number from another.

*Tut*. Simple Subtraction does indeed relate to taking one number from another in such a manner as to show the true difference between them. Addition, you will recollect, is a rule by which numbers are put together, so as to find their whole amount in one sum ; but Subtraction shows how to take one number from another, and consequently is the reverse of addition. In addition I told you, that all small numbers were added in the mind, and their sums committed to memory ; (4. Con. I.) so in subtraction, the most natural way to find the difference between small numbers is to count to the full expression of one of those numbers, which must be the smallest, and then begin with unity, and add one at a time till you have expressed the amount of the largest number ; the number of units which you added will be the difference between the two numbers. In this way the difference between small numbers is obtained and committed to memory. But, as in addition, this method of subtracting is extremely tedious and impracticable for large numbers ; therefore to avoid this, we have the following rule for performing questions in

---

* Addition may be proved as follows ; draw a line underneath the top line of figures ; add together all the figures below the line, and write their sum beneath the sum of the whole ; then add together the top line and the sum of the other lines, and if this sum agrees with the sum of the whole, the work is right ; otherwise it is wrong, and must be looked over again.

## SIMPLE SUBTRACTION.

**Write the greater number, and directly under it write the less, so that units may stand under units, tens under tens, hundreds under hundreds, &c. and draw a line underneath.**

**Begin with the right hand figure of the lower number, and find the difference between it and the figure above it, setting the result underneath.**

**If the figure in the lower number or subtrahend* is larger than the figure over it, borrow ten from the next figure towards the left in the minuend, and add to the figure which is too small ; then write this sum and the lower figure as before, and carry one to the next figure of the subtrahend for the ten which you borrowed.**

**Proceed in this manner with all the figures, and the several differences taken together will be the difference of the two numbers.**

If you wish to find the difference between 464 and 342, you will place them as follows.

$$\begin{array}{r} 464 \\ 342 \\ \hline 122 \end{array}$$

---

* *The larger number is* called the *minuend*, and the smaller *the subtrahend.*

Here you begin on the right, and take the difference between 2 and 4, which is 2, which set underneath; then take the difference between 4 and 6, and between 3 and 4, all of which, read together, is the difference of the two numbers.

If it be required to find the difference between 86032, and 65264, you have another part of the rule to consider.

8 6 0 3 2 minuend.
6 5 2 6 4 subtrahend.

---

2 0 7 6 8 remainder.

In this example you begin as before, at the right, and seek the difference between 4 and 2; but here the lower figure is larger than the upper figure, and it is for such cases that the rule directs to borrow ten. In this case you find that you cannot take 4 from 2; but as 1 in the place of tens, where 3 is, is equal to 10 in the place of units, you must take 1 from 3, and reduce it to units, or divide it into ten parts, which, with the 2 make 12, so that the 2 is changed into 12, from which 4 can be subtracted, leaving 8 for a remainder or difference. Now as you borrowed 1 from the place of tens, viz. from 3, you must consider this one less than it is; for, having taken 1 from it, it must be 1 less. But as it is more convenient to consider the figure under it 1 larger than it really is, you may carry 1 to that figure, and subtract as before. In this example, when you come to subtract 6 from 3, you find that you must borrow ten again, which you are to do from the next figure. But here the next figure is a cipher, and ten cannot be borrowed from it; in this case you borrow from the next figure towards the left, viz. 6, but 6 is in the place of thousands, and 1 from that is equal to 1000, and you do not want but 100, for 0 is in the place of hundreds, and had you borrowed from that place it would have been 100.

*Pup.* You say that I borrow ten, and then carry one to the next figure, for the ten which I borrowed; and that if I had borrowed from 0 it would have been 100, the sum wanted; I do not understand it.

*Tut.* *When* you borrow ten, you do not in reality *borrow but* one, for you take one from a higher, and reduce it to the next lower denomination, which makes ten of it; and as you borrowed but one from the higher denomination, you have but one to carry to it. As to borrowing 100, as in the example, you must consider that 6 is really more than 6; it is in the place of tens, and is 60, and the 1 which is to carry to it makes it 70. The three above is thirty, and you cannot subtract 70 from 30, but must borrow 100 and add to it; yet because the numbers increase in a ten-fold proportion, it is called borrowing ten, however large the number really is. And here, as 0 is in the place of hundreds, the 100 must be borrowed from 6, and 1 from 6 being equal to 1000, 100 must be taken from the 1000, and the 900 remaining are considered as placed at the right of 6, so that the minuend would read thus, 86,900; but as 6 was made less by 1, it must be called 5, so that setting aside the two right hand figures of each given number, they will stand thus.

$$\begin{array}{r} 8\ 5\ 9 \\ 6\ 5\ 2 \\ \hline 2\ 0\ 7 \end{array}$$

You have now seen the whole process of Subtraction, which you must make perfectly familiar.

*Pup.* I think I understand the principles of this rule, and shall be able to answer any question you may ask me.

*Tut.* You may, in order to render what I have said, familiar, perform the following questions.*

What is the difference between 16844 and 9786?
Ans. 7058.

What is the difference between 124682 and 113465?
Ans. 11217.

I will now examine you in the preceding rules, and see if you understand them; if you do not, you must review them, and make every part perfectly familiar.

---

* Subtraction is proved by adding the remainder and subtrahend together; if their sum agrees with the minuend, the work is right.

## *Questions.*

What is arithmetic ?
Of how many kinds is it ?
What is numeration ?
How do numbers increase from right to left ?
When you wish to express any number of tens, what do you place at the right of the significant figure ?
When you wish to express any number between any number of hundreds and tens, how do you do it ?
What is simple addition ?
How do you place the numbers to be added ?
Which column of figures do you add first ?
What is the answer called ?
How do you write down the sum of each column ?
Why do you carry for ten rather than for any other number ?
How do you prove addition
What is simple subtraction ?
How many numbers are required tu perform operations in this rule ?
What are these numbers called ?
How are they placed for performing the work ?
When the figure in the subtrahend is larger than the figure above it, how do you proceed ?
How does it appear that borrowing ten and carrying one to the next figure does not give a wrong answer ?
How is subtraction proved ?

## *Examples for Practice.*

1. A man bought a coach and 4 horses; for his coach he gave $200, for the two first horses he gave $300, and for the other two $250; what did they all cost him ?
Ans. $750.

2. From Boston to Worcester is thirty-eight miles; from Worcester to Hartford is sixty-two miles; from Hartford to New-Haven thirty-four miles; from New-Haven to New-York seventy-six miles; how far is it from Boston to New-York ? Ans. 210 miles.

3. There are five numbers; the first is 2617; the second 893; the third 1702; the fourth as much as the *three first;* the fifth twice as much as the third and *fourth; what is* the whole sum ? Ans. 24252.

4. A general has an army composed of three divisions; the first division consists of 450 men; the second of 375; the third of 560; he wishes for a reinforcement equal to the first and third divisions; what was the strength of his army before and after the reinforcement? Ans. before, 1385; after, 2395.

5. The sum of two numbers is 64892; the greater number is 46234; what is the smaller number? Ans. 18658.

6. A, holds a note against B, to the amount of $1200; B pays $675; what was then due? Ans. $525.

7. A owed B $4850; at one time he paid $200; at another time $475; at another time $40; at another time $1200; and at another time 156; what was then due? Ans. $2779.

8. From the creation to the departure of the Israelites from Egypt, was 2513 years; to the siege of Troy, 307 years more; to the building of Solomon's temple, 180 years; to the building of Rome, 251 years; to the expulsion of the kings from Rome, 244 years; to the destruction of Carthage, 363 years; to the death of Julius Cæsar, 102 years; to the Christian era, 44 years; how long must the world stand after 1823, to make another number of years just equal to the number from the creation to the Christian era? Ans. 2181 years.

## CONVERSATION II.

### SIMPLE MULTIPLICATION AND SIMPLE DIVISION.

*Tut.* Can you tell what simple multiplication is?

*Pup.* I cannot, unless it is what the name implies, a method of multiplying or repeating a number.

*Tut.* Simple multiplication is a rule by which a number is repeated a given number of times. This may be done by addition; if it were required to multiply 12 by 4, we might add 12 to itself three times, and obtain the answer thus:

1 2
1 2 once,
1 2 twice,
1 2 three times,
———
4 8

From this you see that a number may be multiplied any number of times by means of addition. But if it were required to multiply 12 by 16, this would be a tedious manner of doing it. To avoid this we have a rule for repeating a number a given number of times; but before you learn this rule you must know the names by which the several numbers are called, and commit the Multiplication Table to memory.

Multiplication implies three numbers, viz. the number to be repeated, which is called the *Multiplicand*; the number which shows how many times it is to be repeated, which is called the *Multiplier*; and the result of the operation, which is called the *Product*.

The *Multiplicand* and *Multiplier*, considered together, are called the *factors* of the product.

You must now commit the following table to memory; without it you will not be able to make any progress in multiplication.

MULTIPLICATION TABLE.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 | 36 |
| 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 | 44 | 48 |
| 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
| 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 | 60 | 66 | 72 |
| 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63 | 70 | 77 | 84 |
| 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72 | 80 | 88 | 96 |
| 9 | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81 | 90 | 99 | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99 | 110 | 121 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

The invention of this table is ascribed to Pythagoras, and is formed in the following manner: The numbers

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, are written first on the same line, then each of these numbers is added to itself and written in the second line, which contains each number of the first line doubled. Each number of the second line is then added to the number over it, in the first line, which gives the triple of the first, and is the third line; in this manner the table is formed throughout.

To commit this table to memory, begin with the 2 in the left hand column, and multiply each figure in the top line by the 2, and you will find the product in the second line from the top, directly under the figure you multiply. Then take the three in the left hand column, and multiply as before; and in this manner learn the whole table. It is not necessary, however, to learn at first, any further than to 9 in the left hand column.

You must now learn the rule for

## SIMPLE MULTIPLICATION.

**Place the numbers under each other, the largest uppermost, so that units may stand under units, tens under tens, &c. and draw a line beneath.**

**Begin with the unit figure of the multiplier, and multiply each figure of the multiplicand by it, setting the excess of tens in the product of the two figures, under the line, till you multiply the last figure of the multiplicand, when you write the whole product. Then multiply the multiplicand by the next figure of the multiplier, setting the product one place further to the left than the preceding product. Proceed in this manner with all the figures of the multiplier, setting each successive product one place further to the left than the preceding; then the sum of all these products will be the product of the whole.**

If it be required to multiply 38 by 24, the numbers must stand thus:

$$\begin{array}{lrl} \text{Factors,}\left\{\begin{array}{l}\\ \\ \end{array}\right. & \begin{array}{r} 3\ 8 \\ 2\ 4 \end{array} & \begin{array}{l}\text{multiplicand,}\\ \text{multiplier,}\end{array} \\ \cline{2-2} & 1\ 5\ 2 & \\ & 7\ 6\ \ & \\ \cline{2-2} & 9\ 1\ 2 & \text{product.} \end{array}$$

*Pup.* I think I understand the reason for carrying for every ten, but do not see why each successive product* is placed further to the left than the preceding; I should like to have you give the reasons.

*Tut.* In order that you may understand every part, and see the reasons plainly, I will take the following numbers, and multiply them together, explaining every part.

$$\begin{array}{cccc} 1 & 3 & 4 & 8 \\ & 3 & 2 & 4 \\ \hline 5 & 3 & 9 & 2 \end{array}$$

Here I begin with the figure 4 in the multiplier, and first multiply the 8 in the multiplicand by it, the product of which is 32; in 32 there are 3 tens and 2 over, therefore set down 2 and reserve 3 for the product of the next figure in the multiplicand. 4 times 4 are 16, and 3 to carry are 19, which contains ten and 9 over; and in this manner each figure of the multiplicand is multiplied by the 4.

$$\begin{array}{ccccc} & 1 & 3 & 4 & 8 \\ & & 3 & 2 & 4 \\ \cline{2-5} & 5 & 3 & 9 & 2 \\ 2 & 6 & 9 & 6 & \end{array}$$

2. When I multiply by 2, I do not obtain the product in units, as with the 4, but in tens, for 2 is in the place of tens, and therefore the product will be in tens; and this is the reason of placing the product by 2 further to the left than the preceding, as in the above example.

To make this appear more plainly, I will decompound the multiplier. In the multiplier there are 300, 20 & 4; in the first place I multiply by 4, which gives a product in units; I then multiply by 2, which you see is in reality 20. Now as 2 is in the place of tens, the product of the multiplicand by it will be in tens, and must be put in the place of tens. When I multiply by 3 I get the product in hundreds, because 3 is in the place of hundreds, consequently the product of this figure must be carried one place further to the left than the preceding. I will now give you the work in its decompounded state as follows:

---

* *By product* here is meant the product of each figure of the *multiplier by the* multiplicand.

$$\begin{array}{r}1348\\4\\\hline 5392\end{array}\quad\begin{array}{r}1348\\20\\\hline 26960\end{array}\quad\begin{array}{r}1348\\300\\\hline 404400\end{array}\quad\begin{array}{r}5392\\26960\\404400\\\hline 436752\end{array}$$

$$\begin{array}{cccccc} & & 1 & 3 & 4 & 8\\ & & & 3 & 2 & 4\\\hline & & 5 & 3 & 9 & 2\\ & 2 & 6 & 9 & 6 & \\ 4 & 0 & 4 & 4 & & \\\hline 4 & 3 & 6 & 7 & 5 & 2\end{array}$$

Here you will see the reason of placing each successive product one place further to the left than the preceding. You must make this familiar, for it is important to understand it thoroughly before you proceed further.

*Pup.* Does it never happen that the multiplier is larger than the multiplicand?

*Tut.* This never happens, because either of the numbers may be used for a multiplier, and the largest number is always placed uppermost. In multiplying one number by another, the product will be the same, whether you use one or the other number for a multiplier, the truth of which you can see in the following numbers:

$$\begin{array}{ccc} & 3 & 6\\ & 2 & 4\\\hline 1 & 4 & 4\\ 7 & 2 & \\\hline 8 & 6 & 4\end{array}\qquad\qquad\begin{array}{ccc} & 2 & 4\\ & 3 & 6\\\hline 1 & 4 & 4\\ 7 & 2 & \\\hline 8 & 6 & 4\end{array}$$

It does indeed happen that the largest number is used for a multiplier. When there are ciphers at the right of either number, they may be omitted in multiplying, and annexed to the product; therefore when the largest number has ciphers at its right, and the significant figures do not exceed those of the smallest number, the ciphers may be omitted and this number taken for a multiplier, as with the following numbers, 4800 and 724.

$$\begin{array}{r} 7\;2\;4\phantom{\;0\;0} \\ 4\;8\;0\;0 \\ \hline 5\;7\;9\;2\phantom{\;0\;0} \\ 2\;8\;9\;6\phantom{\;0\;0\;0} \\ \hline 3\;4\;7\;5\;2\;0\;0 \end{array}$$

Again, that you may see the nature of the work as plainly as possible, I will give another example.

Multiply 24689 by 4.

$$\begin{array}{rcrcrcr} 2\;4\;6\;8\;9 & & & & & & \\ 4 & & & & & & \\ \hline 3\;6 & = & 4 & \times & 9 & = & 36 \\ 3\;2\phantom{\;6} & = & 4 & \times & 80 & = & 320 \\ 2\;4\phantom{\;6\;6} & = & 4 & \times & 600 & = & 2400 \\ 1\;6\phantom{\;6\;6\;6} & = & 4 & \times & 4000 & = & 16000 \\ 8\phantom{\;6\;6\;6\;6} & = & 4 & \times & 20000 & = & 80000 \\ \hline 9\;8\;7\;5\;6 & = & 4 & \times & 24689 & = & 98756 \end{array}$$

3. In this example you see that the multiplicand is decompounded into 20,000, 4,000, 600, 80 and 9, each of whose products is placed separately, which shows plainly the reason for placing the products as directed.

When there are ciphers in the middle of the multiplier, *they may be omitted, carrying the product of the next figure two, three, &c. places to the left according to the number of ciphers, thus:*

$$\begin{array}{r} 1\;2\;4\;6 \\ 2\;0\;4 \\ \hline 4\;9\;8\;4 \\ 2\;4\;9\;2\phantom{\;0\;0} \\ \hline 2\;5\;4\;1\;8\;4 \end{array}$$

When either of the factors is a unit with ciphers at its right, *place the ciphers at the right of the other number, and it will be the product of the two numbers.*

Multiply 2486 by 1000. Ans. 2486000.

Every removal towards the left increases the value of the figures ten times, and here 2486 is to be repeated 1000 times, which is done by removing it three places to the left.

*Pup.* I think I now understand Multiplication sufficiently to answer any questions which may be put in this rule, and should like to apply the rule by performing a few examples.

*Tut.* You may multiply together the following numbers.

Multiply 5943267 by 42. Ans. 249617214.
Multiply 24689468 by 90. Ans. 2222052120.
Multiply 35012 by 100. Ans. 3501200.

◆

Having now learned how to multiply numbers together, so that one number may be repeated a given number of times, and the amount of these repetitions expressed in one sum, you will want to know how to bring this number into its former state, viz. to divide it into a given number of parts. And as multiplication can be performed by addition, so division can be performed by subtraction, for multiplication and division are the reverse of each other, the same as addition and subtraction are. To reverse the example given in multiplication (1. Con. II.) we should have to find how many times 12 is contained in 48; this may be done by subtraction in the following manner:

48
12 once,

36
12 twice,

24
12 three times,

12
12 four times.

00

Here by subtracting 12 from 48 you decrease 48 by 12 and then, again subtract 12 from the remainder, and continue to subtract till you can subtract no longer. Then the number of subtractions you made will be the number of times that 48 contains 12. Thus in dividing 48 by 12 you subtract 12 four times, and 4 times 12 are 48. But if it were required to divide 288 by 12, this process would be very long and tedious. To remedy this inconvenience we have the following rule for performing

## SIMPLE DIVISION.*

Write the dividend, and on the right and left of it draw a curved line. Place the divisor at the left of the dividend. Seek how often the divisor is contained in just so many of the left hand figures of the dividend as are necessary, and set the result on the right of the dividend, for the first figure of the quotient.

Multiply the divisor by the quotient figure, and place the product under those figures of the dividend which were used for obtaining the quotient figure. Subtract this product from the dividend, and to the remainder bring down the next figure of the dividend towards the right, which makes a new dividend, which must be proceeded with as was the first; and proceed in this manner till you have brought down all the figures of the dividend.

If it were required to divide 248 by 16, the work would stand thus :—

```
                 dividend.
divisor, 1 6 ) 2 4 8 ( 1 5 quotient.
               1 6
               ---
                 8 8
                 8 0
                 ---
                   8 remainder.
```

*Pup.* The *work* here is plain, but I do not understand the reason of multiplying the divisor by the quotient figure, and subtracting as I do. I should like to have you explain every part to me that I may understand the whole.

---

* Before learning the rule for simple division you must be informed, that the number given to be divided is called the *dividend;* *the number* by which it is divided is called the *divisor;* and the *result or answer to* the question is called the *quotient;* whatever *remains after division* is called the *remainder.*

*Tut.* I am glad that you wish to understand the reasons of the rule, and do not wish to get along merely by doing the sums without knowing any thing else about them, as scholars generally do.

I will now give an example with an explanation of the whole operation. Suppose it were required to divide 64268 by 28; the numbers for the operation would stand thus:—

```
24)64268(
```

4. Here you are in the first place to find how often 24 is contained in 64; 64 here is not merely 64, but 64,000; therefore we will suppose the dividend to be 64,000, and divide it by 24.

```
24)64000(2
   48
   --
   16
```

Here we find that 64 contains 24 twice, and 16 over; but as 64 signifies 64,000, so 16 must signify 16,000; and the 2 in the quotient does not signify 2 merely, but 2,000, and the product of 2,000 by 24 is 48,000, which 48 in the work signifies; and 48,000 subtracted from 64,000 leaves 16,000, which is expressed by the 16 in the work. We have now found that 64,000 contains 24 two thousand times, and 16,000 remainder; now had the given dividend been only 64,000, the next step would be to find how often 16,000 contains 24. But the given dividend is 64,268; therefore we have 200 to add to 16,000, because 2 in the dividend is in the place of hundreds, and is 200. We will consider the dividend to be 64,200, and the work will stand thus:—

```
24)64200(26
   48
   --
   162
   144
   ---
    18
```

Here we seek how often 24 are contained in 162, and find it is 6 times; and as 162, the new dividend, is really 16,200, 6 must be 600, which multiplied by 24 gives *14,400, and this* subtracted from 16,200 leaves 1800, which the remainder 18 expresses. Now had the dividend been only 64,200, the next step would be to find how often 1800 contains 24; but the dividend being 64,268 we have 60 to add to 1800, for 6 in the dividend expresses 60. We will now consider the dividend to be 64,260, and the work will stand thus :—

```
24)64260(267
   48
   ---
   162
   144
   ---
    186
    168
    ---
     18
```

We again seek how often the divisor is contained in the dividend, which is now reduced to 1860, which 186 expresses, and find that 24 is contained in 186 seven times; but 186 is really 1860, so that 7 must be 70; 70 times 24 are 1680, which 168 expresses, and 1680 subtracted from 1860 leaves 180, which 18 expresses. Had the given dividend been but 64260, we should now have only to find how often 180 contains 24; but the dividend is 64268, therefore we have 8 to add to 180. We will now consider the dividend as it was given, viz. 64268, and the work is as follows :—

```
24)64268(2677
   48
   ---
   162
   144
   ---
    186
    168
    ---
     188
     168
     ---
      20
```

*Pup.* *I* think I now understand every part of this work *completely*, and should like some examples that I may *render the* rule familiar.

*Tut.* Before I give any examples I will direct you how to proceed when the divisor is any number not over 12.

5. When the divisor does not exceed 12, the division may be performed without writing the product of the quotient and divisor. *First find how often the divisor is contained in the left hand figures as before: then subtract in your mind the product of the quotient figure and the divisor, from those figures of the dividend used, and consider the remainder as prefixed to the next figure which will form another dividend, with which proceed as before. Proceed in like manner with all the figures of the dividend, setting the quotient under the dividend.*

4 ) 4 2 6 8 5 6

1 0 6 7 1 4

Here I begin with 4 in the dividend, and find 4 is contained in it once, which I set underneath; then I seek how many times 4 are contained in the next figure, which being smaller than the divisor, I set down 0, and seek how often 4 is contained in 26, and find it is 6 times and 2 over; then after setting down the 6, I consider the 2 placed at the left of 8, which makes 28; then I seek how often 4 is in 28, and proceed in this manner with all the figures.

| | |
|---|---|
| Divide 8649 by 6. | Ans. $1441\frac{1}{2}$. |
| Divide 6246963 by 8. | Ans. $780870\frac{3}{8}$. |
| Divide 84656 by 12. | Ans. $7054\frac{2}{3}$. |
| Divide 3211473 by 27. | Ans. 118943. |
| Divide 1406373 by 108. | Ans. 13021. |

6. When there are ciphers at the right of the divisor, cut them off, and cut off as many figures from the right of the dividend. Then proceed as when there are no ciphers, and the quotient will be the same as if the ciphers had not been cut off. The figures cut off at the right of the dividend must be considered as a remainder, and placed at the right of the other remainder.

Divide 14286 by 2400.

2 4, 0 0 ) 1 4 2, 8 6 ( 5
1 2 0

2 2 8 6 remainder.

From what you have been taught you will readily perceive, that Multiplication and Division prove each other. If you wish to prove Multiplication, divide the product by one of the factors, and the quotient will be the other factor. If you wish to prove Division, multiply the quotient and divisor together, and if there is a remainder, add it to the product, and you will have the dividend.

Divide the following numbers, and prove the work.

```
18)68573(3809          3809
   54                    18
   --                  ------
   145                 30472
   144                 3809
   ---                     11
     173               ------
     162               68573
     ---
      11
```

* * *

### *Questions.*

What is Simple Multiplication?
How many numbers are required to perform an operation in this rule?
What are these numbers called?
What is the answer called?
How must the factors be placed for Multiplication?
Why do you place the right hand figure of each successive product one place further to the left than the preceding?
When there are ciphers on the right of the significant figures, how do you proceed?
How is Multiplication proved?
What is Simple Division?
How many numbers are given to perform an operation?
What are these numbers called?
How do you place them for the operation?
How does it appear that, by multiplying the divisor by the quotient figures, and subtracting their products from each successive dividend, you get the right answer?

When there are ciphers on the right of the divisor, why does it not give a wrong answer to cut them off, if you cut off the same number of figures from the dividend?

How is Division proved?

*Examples for Practice.*

1. Multiply 684389 by 156. Ans. 106764684.
2. Multiply 9465287 by 48964. Ans. 463458312668.
3. Multiply 82164973 by 3027. Ans. 248713373271.
4. Multiply 8496427 by 874359. Ans. 7428927415293.
5. Divide 49561776 by 5137. Ans. 9648.
6. Divide 4637064283 by 57606. Ans. 80496.
7. Divide 293839455936 by 8405. Ans. 34960078.

8. Four men enter into partnership; each man puts in $4060; What is the whole amount of stock? Ans. $16240.

9. If one acre of land will produce 88 bushels of corn, how much will 168 acres produce? Ans. 14784.

10. What sum of money must be divided among 18 men so that each man may receive $112? Ans. 2016.

11. A vessel which had a crew of 875 men captured another vessel, which gave each man $245, prize money; what was the value of the ship? Ans. $214375.

12. If a man travel 35 miles in one day, how far will he travel in six weeks and three days, allowing six days to a week? Ans. 1365.

13. If a man spend $4, per day, how much will he spend in one year, there being 365 days in a year? Ans. $1460.

14. A general has an army consisting of 16 regiments; each regiment consists of 12 companies, and each company, of 3 officers, 8 non-commissioned officers, 2 musicians, and 64 privates; he has an order to detach 6 non-commissioned officers and privates from each company; after the detachment he has to meet an enemy; how many men has he? Ans. 13824.

15. A man raised 64562 bushels of corn on 1565 acres; how many bushels was that per acre? Ans. 41.

16. What number multiplied by 246 will give 2429629578? Ans. 9876543.

17. There is an estate of $16896 to be divided among 8 heirs; how much is each share? Ans. $2112.

18. If a man's income be $12060 per year, how much is it per day? Ans. $33.

19. What number must I multiply by 80496 that the product may be 4637064283? Ans. 57606.

20. A man bought a farm of 425 acres, for which he gave $6840; what was that per acre? Ans. $16$\frac{40}{425}$.

21. Four men fitted a fleet of 6 vessels for sea; the first vessel and cargo were worth $1648; the second double the first; the third double the first and second; the fourth was worth as much as the first and third; the fifth was worth as much as the fourth and half the third; the sixth was worth as much as the first and fifth; the fourth vessel and cargo were lost at sea; the others returned, and were sold for three times as much as the whole cost; what was each man's share of the proceeds from the sale? Ans. $45732.

## CONVERSATION III.

### VULGAR AND DECIMAL FRACTIONS.

*Tut.* What do you think Vulgar Fractions are?

*Pup.* I cannot tell unless they are what the name implies, broken parts of a whole.

*Tut.* You have formed a very correct idea of them; they are truly parts of a whole, or integer.

1. In division you must have observed that frequently after you had got all the quotient figures, there would be a remainder. This remainder is a fraction. If you divide 17 by 5 you will have 3 for a quotient, and 2 *remainder.* This 2 shows the number of units there are in 17 *over and above* 3 times 5. Had 20 been used instead of

17, there would have been no remainder, because 4 times 5 are 20; and had 15 been used instead of 17, there would have been no remainder.

The divisor, in division, expresses the number of units, which one of those parts contains, which a unit of the quotient expresses In the above example, 5 shows the number of units which a unit or one in the quotient expresses, for 3 in the quotient expresses 3 times 5, because 5 is the divisor, and is contained 3 times in 17. Now in dividing 17 by 5, there is a remainder of 2, which shows how many units there are left, after taking the divisor from the dividend a given number of times. In the example above, the 2 which remains shows that after taking 5 from 17, 3 times, there are 2 fifths of another 5. This remainder or fraction is expressed thus, $\frac{2}{5}$, so that the quotient will read $3\frac{2}{5}$, three and two fifths. Every fraction, then, consists of two numbers; the first, which shows how many parts of a whole the fraction expresses, is called the *numerator;* the second, which shows how many of these parts it takes to make a whole, is called the *denominator;* because the denomination of the fraction is deduced from it.

*Pup.* Then I suppose there are a great number of denominators.

*Tut.* There are. There is no number which may not be used both for a *numerator* and *denominator.*

*Pup.* If the terms are very large, is it not many times difficult to obtain their value?

*Tut.* It would be sometimes if we could not reduce them to less numbers; for the terms may be reduced to other numbers and retain the same value.

*Pup.* I cannot see how a number can be reduced, and not alter its value.

2. *Tut.* Suppose you have $\frac{4}{12}$, and you wish to reduce it to lower terms. If you divide each term by 4, you will retain the same value, although the figures are altered, for $\frac{4}{12}$ are just equal to $\frac{1}{3}$, 4 being one third of 12. Thus the fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, $\frac{6}{12}$, &c. are equal

to each other, although the terms are different, for in each fraction the denominator is just double the numerator. In $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, $\frac{4}{12}$, &c. the denominators are triple the numerators, and consequently the fractions are equal to each other. Now if any of these fractions are to be reduced, you will divide the numerator and denominator by the same number; and as these fractions are equal to each other, they can all be reduced to the same fractional terms. Thus $\frac{2}{4}$ may be reduced to $\frac{1}{2}$ by dividing both terms by 2; $\frac{3}{6}$ by dividing them by 3; $\frac{3}{9}$ may be reduced to $\frac{1}{3}$ by dividing both terms by 3; $\frac{4}{12}$ by dividing them by 4, and the same with the others.

To make this familiar to you I will give you a few examples.

Reduce $\frac{25}{75}$ to its most simple terms. Ans. $\frac{1}{3}$.

Reduce $\frac{160}{168}$ to its most simple terms. Ans. $\frac{20}{21}$.

Reduce $\frac{2640}{2880}$ to its most simple terms. Ans. $\frac{11}{12}$.

Hence it follows, that in order to simplify a fraction we must divide its two terms by one of the numbers 2, 3, 4, &c.; but by this mode of proceeding it will often be necessary to perform a great number of operations.

3. Here it may be proper to mention, that dividing the numerator lessens the fraction, and dividing the denominator increases it. If you divide the numerator in $\frac{4}{12}$ by 4, you make it $\frac{1}{12}$. If you divide the denominator by 4, you increase it, making it $\frac{4}{3}$. Multiplying the numerator increases the value of the fraction; multiplying the denominator decreases its value. If you multiply the denominator in $\frac{4}{12}$ by 4, you make it $\frac{4}{48}$; if you multiply the numerator you make it $\frac{16}{12}$.

We will now consider the general rule for finding a number by which the two terms of the fraction can be reduced to their lowest terms. This number is called the common divisor or common measure.

**If there be two numbers only, divide the greater by the less, and this divisor by the remainder, and so on till nothing remains, *always* dividing the last divisor by the last remainder; then will *the last divisor* be the greatest common divisor required.**

When there are more than two numbers, find the greatest common divisor of two of them first; then of that common divisor and one of the other numbers, and so on with all the numbers; when the greatest common divisor last found will be the answer.

If 1 happen to be the common divisor, the numbers are prime to each other, and are incommeasurable, or in their lowest terms.

What is the greatest common divisor of 132 and 356?

```
132)356(2
    264
    ---
     92)132(1
         92
         --
         40)92(2
            80
            --
            12)40(3
               36
               --
                4)12(3
                  12
                  --
```

*Pup.* I do not understand why dividing in this way should give a common divisor; I should like to have you explain it.

4. *Tut.* It is plain that the greatest common divisor of these two numbers cannot exceed the least, 132; then it is proper to try if 132, which divides itself and gives 1 for a quotient, will divide 356; if it will, and not leave a remainder, 132 is the greatest common divisor of those two numbers. But in this example it is not the case; there is a remainder of 92.

Now it is evident, that every common divisor of the two numbers 132 and 356 should also divide 92, the remainder resulting from their division; because the greater, 356, is just equal to twice 132 with the remainder added. Now the common divisor will divide both numbers without a remainder, and as it will divide 132, 356, and 92, it follows that by dividing 132 by the remainder, after dividing 356 by 132, and continuing to divide in this manner, you will get a common divisor, which will divide both the terms, 132 and 356, and each of the remainders with exactness.

Thus, in the example given, 4, the greatest common divisor of 132 and 356, will also divide each of the remainders, 92, 40, and 12.

Having thus explained the rule, you will be able, if you have given proper attention to it, to perform the operations of the rule without difficulty.

*Pup.* The reasons appear very plain, and I think I shall be able to apply the rule in all cases; it is not so difficult as I at first imagined.

*Tut.* You may find the greatest common divisor for the following numbers.

What is the greatest common divisor of 24 and 36? Ans. 12.

What is the greatest common divisor of 35 and 100? Ans. 5.

What is the greatest common divisor of 1224 and 1080? Ans. 72.

The common divisor of more than two numbers is found by the common divisor of the two numbers, and one of the other numbers, and so on with the whole, the last greatest common divisor being the greatest common divisor of all the numbers.

It is not always necessary to find the greatest common divisor, in order to reduce the terms of the fraction, for some numbers are such that they may be known to be divided by certain numbers as their common divisors. Every number terminated by 0, 2, 4, 6, or 8, is divisible by 2, for in dividing any number by 2, only 1 can remain from the tens; and as no figure prefixed to any of these will make it such a number that, when divided by 2, 1 will remain, it follows that it must be divisible by 2.

Likewise every number terminated on the right by 0 or 5, is divisible by 5, for when the division by tens has been performed, the remainder, if there is any, must be 1, 2, 3, or 4; the remaining part of the operation will be *performed* on the numbers 0, 5, 10, 15, 20, 25, 30, 35, *40 or 45, all of* which are divisible by 5.

That you may make the rule familiar, you may reduce the following fractions.

Reduce $\frac{48}{272}$ to its least terms. Ans. $\frac{3}{17}$.
Reduce $\frac{192}{576}$ to its lowest terms. Ans. $\frac{1}{3}$.
Reduce $\frac{252}{364}$ to its least terms. Ans. $\frac{9}{13}$.
Reduce $\frac{6144}{8192}$ to its least terms. Ans. $\frac{3}{4}$.

You have now been taught how to reduce fractions to their lowest terms; the next step will be to learn how to multiply fractions.

---

5. In Multiplication, when the multiplier is a whole number, it shows how many times the multiplicand is to be repeated. (2. Con. II.) But the term multiplication, when applied to fractional numbers, does not always imply augmentation, as it does in whole numbers. When it is required to multiply by 4, the multiplicand is 4 times repeated, and consequently the product is 4 times greater than the multiplicand; if it were required to multiply by 2, which is one half of 4, the product would be one half of what it would have been, had the multiplier been 4; and if it be required to multiply by 1, the product will be one half of the product by 2, or merely a repetition of the figures of the multiplicand; thus the smaller the multiplier, the smaller the product will be. $\frac{1}{2}$ bears the same proportion to 1, that 1 does to 2; therefore if you multiply by $\frac{1}{2}$, the product will be one half what it would have been had you multiplied by 1, viz. one half the multiplicand.

If, for instance, it be required to multiply any number by $\frac{2}{3}$, all you want is two thirds of that number. This is obtained by taking one third of it, and multiplying it by 2. Thus to multiply 24 by $\frac{2}{3}$, you first find what $\frac{1}{3}$ is by division, and then repeat that twice, and it will give the product of 24 by $\frac{2}{3}$. Thus you see that to multiply any number by $\frac{2}{3}$ is to take from it such a part as shall be equal to $\frac{2}{3}$ of that number, or equal to twice $\frac{1}{3}$.

When two mixed numbers are to be multiplied together it is best to reduce them to improper fractions, and then proceed as with proper fractions. Thus to multiply $4\frac{1}{2}$ by $3\frac{1}{4}$ they would be $\frac{9}{2} \times \frac{13}{4} = \frac{117}{8}$.

It is usual in expressing fractions of fractions, to read them thus, $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{3}{4}$.

We have now gone through with multiplication of fractions, and will attend to the division of them.

---

8. When we say that one number contains another, we speak correctly, provided the numbers are whole ones; but when applied to fractions, it is not strictly correct, for as multiplying a number by a fraction decreases it, so dividing a number by a fraction increases that number. If it were required to divide 48 by 4, the quotient would be 12; but if the divisor had been 2, which is one half of 4, instead of 4, the quotient would have been 24; and if instead of 2 we had divided by 1, the quotient would have been just the dividend, or double the quotient by 2. Hence it follows that to divide by $\frac{1}{2}$, the quotient must be double the dividend. From what has here been said, and from the relation which division has to multiplication, it appears that the division of fractions must be the reverse of the multiplication of them. Hence the following rule.

To divide one fraction by another, *invert the divisor, and proceed exactly as in multiplication of fractions.* When a whole number is to be divided by a fraction, it may be expressed fractionwise by putting a unit for its denominator; thus 4 is expressed $\frac{4}{1}$, and the same may be done in the *multiplication* of fractions.

| | |
|---|---|
| Divide 9 by $\frac{2}{5}$. | Ans. $\frac{45}{2}$. |
| Divide 8 by $\frac{3}{4}$. | Ans. $\frac{32}{3}$. |
| Divide $7\frac{1}{2}$ by $\frac{1}{3}$. | Ans. $\frac{45}{2}$. |
| Divide $\frac{63}{9}$ by $\frac{9}{63}$. | Ans. 49. |
| Divide $\frac{22}{3}$ by $\frac{22}{3}$. | Ans. 1. |
| *Divide* $\frac{7}{5}$ by $\frac{19}{11}$. | Ans. $2\frac{1}{35}$. |

From what has been said respecting the multiplication and division of fractions, it may be proper to give the following table.

| | | |
|---|---|---|
| By multiplying<br>By dividing | the numerator, the fraction is | multiplied.<br>divided. |
| By multiplying<br>By dividing | the denominator, the fraction is | divided.<br>multiplied. |

*Pup.* You have taught me how to multiply and divide fractions, and have said nothing about adding and subtracting them. Is it not necessary sometimes to add and subtract them?

---

*Tut.* It is. And it may seem preposterous to you that I should teach you how to multiply and divide before teaching you how to add and subtract. But I have done this because a fraction, being a remainder after division, is more immediately connected with multiplication and division, and is more easily performed on, by them, than by addition and subtraction.

9. When it is required to add or subtract fractions, the reasons of the operations are very plain, if the denominators are all alike. If it be required to add $\frac{3}{8}$ to $\frac{5}{8}$ it is evident that adding together the numerators and placing their sum over the common denominator, will give the true amount.

But when the denominators of the several fractions are different, this will not be the case, for the parts of which they are composed are of different magnitudes. But to avoid this difficulty, the fractions are reduced to a common denominator.

If, for instance, it be required to add $\frac{3}{4}$ to $\frac{5}{6}$, we must first reduce them to a common denominator in the following manner.

10. If each term of the first fraction be multiplied by 6, the denominator of the second, the first fraction will be changed into $\frac{18}{24}$; and if each term of the second fraction be multiplied by 4, the denominator of the first, it will be changed into $\frac{20}{24}$, each of them equal to their former expressions.

Again, if it be required to subtract $\frac{3}{4}$ from $\frac{5}{8}$ the same difficulty presents, as when we wished to add them. But being reduced to a common denominator, we can subtract one from the other and obtain the true difference between them.

*Pup*. When there are several fractions to be added or subtracted at the same time, how shall I proceed?

*Tut*. When more than two fractions are given, the process is the same; for all the denominators must be alike, since each one is the product of all the other denominators. Hence we have the following rule.

"Any number of fractions are reduced to a common denominator, by multiplying the two terms of each by the product of the denominators of all the others." Or by multiplying each numerator by all the denominators, except its own, for a numerator, and all the denominators continually together for a common denominator.

Reduce $\frac{3}{4}$ and $\frac{5}{9}$ to a common denominator.

$$\left.\begin{array}{l}3 \times 9 = 27\\ 4 \times 5 = 20\end{array}\right\}\text{new numerators.}$$

$$4 \times 9 = 36 \text{ common denominator.}$$

Reduce $\frac{1}{4}$, $\frac{2}{5}$ and $\frac{5}{8}$ to equivelent fractions, having a common denominator.

$$\left.\begin{array}{l}5 \times 8 = 40 \times 4 = 160\\ 5 \times 4 = 20 \times 8 = 160\\ 4 \times 8 = 32 \times 5 = 160\end{array}\right\}\text{common denominators.}$$

$$\left.\begin{array}{l}40 \times 1 = 40\\ 20 \times 5 = 100\\ 32 \times 2 = 64\end{array}\right\}\text{new numerators.}$$

Ans. $\frac{40}{160}$, $\frac{100}{160}$, $\frac{64}{160}$.

Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$ to a common denominator.

Ans. $\frac{576}{1152}$, $\frac{768}{1152}$, $\frac{864}{1152}$, $\frac{960}{1152}$, $\frac{1008}{1152}$.

Reduce $\frac{2}{10}$, $\frac{3}{5}$, $\frac{4}{7}$ and $\frac{5}{9}$ to a common denominator.

Ans. $\frac{630}{3150}$, $\frac{1890}{3150}$, $\frac{1800}{3150}$, $\frac{1750}{3150}$.

Having now learned how to find a common denominator, you will be able to add and subtract fractions.

*To add fractions, first reduce them to a common denominator, then the sum of all the numerators, written over the common denominator, will form the fraction required.*

When the fractions are mixed numbers, reduce them to improper fractions; and when there are compound fractions, or fractions of fractions, they must be reduced to a simple fraction by multiplying all the numerators together for a numerator, and all the denominators together for a denominator.

| | |
|---|---|
| Add $\frac{3}{7}$ and $\frac{5}{2}$ together. | Ans. $\frac{41}{14}$. |
| Add $\frac{3}{7}$, $\frac{4}{2}$ and $\frac{3}{5}$ together. | Ans. $3\frac{1}{35}$. |
| Add $\frac{3}{5}$, $\frac{4}{5}$ of $\frac{1}{3}$ and $9\frac{3}{2}$ together. | Ans. $10\frac{1}{60}$. |

---

To subtract one fraction from another; *prepare them as in addition; then the difference of the numerators written over the common denominator, will be the difference required.*

| | |
|---|---|
| From $\frac{2}{3}$ take $\frac{1}{3}$. | Ans. $\frac{1}{3}$. |
| From $\frac{13}{20}$ take $\frac{4}{10}$. | Ans. $\frac{1}{4}$. |
| From $5\frac{3}{8}$ take $2\frac{1}{2}$. | Ans. $2\frac{7}{8}$. |
| From $3\frac{7}{9}$ take $2\frac{10}{11}$. | Ans. $8\frac{3}{5}$. |

---

From what you have been taught concerning the increase of numbers in a tenfold proportion, and the intricacy of vulgar fractions, it must appear evident to you, that, if fractions were made to decrease in the same manner that whole numbers increase, or if whole numbers continued their manner of decrease from left to right, to fractions, the labour of operating with them would be much facilitated. We have already a rule for this, which is called

## DECIMAL FRACTIONS.

And can you tell me what decimal fractions are?

*Pup.* I cannot. I did not know before, that there was more than one kind of fractions. I thought, that when a unit could be broken into any number of parts whatever, and that by these parts any part of a unit could be expressed in its true value, and all questions performed by them, that any other kind of fractions would be useless.

*Tut.* Although vulgar fractions can be applied in all cases where they are wanted, and that correctly, yet from the nature of numeration, it must appear plain to you that if fractions were made to decrease in the same manner that whole numbers increase, the labour of performing on fractional numbers would be greatly facilitated, and rendered equally easy with whole numbers. Such is the nature of decimal fractions.

11. As it is a law of whole numbers that they increase from right to *left* in a tenfold proportion, so it is a law of decimal fractions, that they *decrease* to the *right* in a *tenfold proportion.* You have been taught (2. Con. I.) that a unit is the smallest whole number that we have, and that every time we remove it one place to the left, we increase it ten times.

12. Now in order that fractions may be made to correspond with this law, a unit is made to consist of ten parts, and each of these tenth parts to consist of tenths parts or hundredth parts of a unit, and each of these hundredth parts to consist of tenths parts, or thousandths of a unit, and so on, no part ever becoming so small but that it may be divided into ten parts.

| | |
|---|---|
| The unit is equivalent to | 10 tenths; |
| One of these tenths is equivalent to | 10 hundredths; |
| One of these hundredths is equivalent to | 10 thousandths; |

And so on to infinity, one part of the preceding being equal to ten of the next succeeding, *decreasing in the same manner to the right in which whole numbers increase to the left;* or it merely is a continuation of decrease from left to right in the same manner as though *whole numbers and decimals were all alike, and decreased to the right, down to the most minute parts.*

*Pup.* I now think I understand what a decimal fraction is, and it appears to me that they will entirely supersede the use of vulgar fractions: for I cannot see why they may not answer every purpose equally as well, and make the work much more easy.

*Tut.* They will answer all the purposes for which you *will want to* use fractions. But, notwithstanding, it was necessary that you should understand vulgar fractions, or the principles of them, in order to understand decimals; for there is not so much difference between them as you probably imagine. Vulgar fractions always have a denominator expressed; but decimal fractions, having their denominators increase in a uniform ratio, do not require that they should be expressed.

Suppose you wish to write three and half in decimal form, you would do it thus, 3,5, which is read, three and five tenths. This may be changed into a mixed number by expressing the denominator, thus $3\frac{5}{10}$.

If it were required to write 27 and 3 tenths, 40 and 22 hundredths, 12 and 11 thousandths, they should stand as follows,

27,3
40,22
12,011

13. If the fractions were expressed with their denominators they would stand as follows, $27\frac{3}{10}$, $40\frac{22}{100}$, $12\frac{11}{1000}$, so that you will see plainly that there is no need of expressing the denominators to decimals, as their denominators are always 1, with as many ciphers annexed as there are decimal figures, consequently their denominators are always known. You can always reduce a decimal to a vulgar fraction, by placing, for its denominator, a unit with as many ciphers annexed as there are figures in the decimal expression.

That you may be able to enunciate, or reduce to language, numbers with decimal expressions, I will give the following table.

The number 26,4 is read 26 and 4 tenths;
—— 10,02 is read 10 and 2 hundredths;
—— 0,24 is read 24 hundredths;
—— 0,024 is read 24 thousandths;
—— 0,246 is read 246 thousandths;
—— 0,0000042 is read 42 ten millionths;
—— 0,5 is read 5 tenths;
—— 0,50 is read 50 hundredths;
—— 0,500 is read 500 thousandths.

From this table you will see that it makes no difference in the value of decimals, whether ciphers are placed at their right or not; for 0,5 bears the same proportion to its denominator that 50, 500 and 5000 do to theirs. (2. Con. III.)

*Pup.* From this explanation of decimal fractions, I cannot see as operations with them are performed any way different from operations with whole numbers.

14. *Tut.* All the difference, in general, will be in pointing, for the decimals must be separated from the whole numbers by a comma, or you cannot tell how many decimals there are. And you must know what the value of the figures is separately, or you cannot tell what it is collectively. Ciphers placed at the left of decimals decrease their value in tenfold ratio. Thus decimals are governed by the same law of increase and decrease that whole numbers are in regard to the operations that are to be performed by them.

To add numbers accompanied by decimals;—

Place the numbers so that the commas may stand perpendicularly one under the other. Add as in whole numbers, and point off so many places for decimals, as shall be equal to the greatest number of decimals in any of the numbers added.

Add 126,428, 69,0327 and 12,004 together.

$$\begin{array}{r@{,}l} 126 & 428 \\ 69 & 0327 \\ 12 & 004 \\ \hline 207 & 4647 \end{array}$$

Add 426,342, 6,05, 92,0001 and 14 together.
Ans. 538,3921.

The rule for the subtraction of whole numbers will *apply to the* subtraction of decimals, as well as the rule *for the* addition of whole numbers to the addition of decimals.

Hence, to subtract decimals ;—

Write the larger number, and directly under it write the smaller, so that the comma of the latter may be exactly under the comma of the former. Then proceed as in whole numbers, pointing off so many places for decimals as shall be equal to the greatest number of decimals in either of the given numbers.

Subtract 0,246231 from 0,468.

$$\begin{array}{r} 0,468\phantom{231} \\ 0,246231 \\ \hline 0,221769 \end{array}$$

Here you will perceive that the number to be subtracted consists of more figures than the other number ; but as ciphers, placed at the right of decimals, do not alter their value, so the subtrahend in this case is not larger in value than the minuend, although it contains more figures. The minuend here is 468 thousandths, and the subtrahend is 246231 millionths. But 468 thousandths can be reduced to millionths without altering its value, thus,

$$\begin{array}{r} 0,468000 \\ 0,246231 \\ \hline 0,221769 \end{array}$$

From 42,0348 take 26,0082. Ans. 16,0266.

From 9 take 0,864268. Ans. 8,135732.

15. In the multiplication of decimals, the only variation from the manner of multiplication of whole numbers is in pointing the product. The comma in the expression of a whole number with decimals, shows the place where the whole numbers and decimals begin. By removing the comma towards the right, figures, which composed the fractional part, are made to represent whole numbers, and in this manner the given number is increased. On the contrary, if the comma is removed towards the left,

the figures, which before represented whole numbers, are made to represent fractions; and thus the value of the given number is decreased. In the expression, 124,682, if you remove the comma to the right two places, the expression will read 12468,2; the units will have become hundreds, the tenths tens, the hundredths units, the thousandths tenths. But if the point is removed towards the left two places, 1,24682, the expression is decreased in value; the tens will have become tenths, the units hundredths, the tenths thousandths, &c. from which you must be sensible of the importance of keeping the comma in its proper place. We will first consider the multiplication of decimals by whole numbers.

16. Suppose it were required to multiply 0,5 by 2, the product would be 10, which is 10 tenths. 0,5 is 5 tenths, equal to one half, which multiplied by 2 gives a product of 1; consequently the product of 0,5 by 2 is 1, (5. Con. III.) Again, if it were requested to multiply 42,6 by 3, you would begin as in whole numbers, and multiply 6 by 3, the product of which is 18, and 6 being tenths, 18 must be tenths, and as 10 tenths are equal to 1 in the place of units, you must carry 1 to the unit figure, and set down the excess of tens.

The product of 42,6 by 3 is 127,8; now if you strike out the comma, you reduce this number to tenths, and as the product is in tenths, by dividing it by 10, you reduce it to units; and dividing by 10 is merely cutting off the right hand figure; and whatever the decimal expression is, whether tenths, hundredths, or thousandths, &c. the product will be in the same value, and must be divided by 10, 100, 1000, &c.

From what I have said, you will see plainly that—

To multiply any number, accompanied by decimals by a whole number, multiply the numbers together as though they were all whole numbers, and cut off so many figures from the right of the product as there are decimals in the factor.

| | |
|---|---|
| Multiply 246,02 by 8. | Ans. 1968,16. |
| Multiply 32,1509 by 15. | Ans. 482,2635. |
| *Multiply* 0,840 by 840. | Ans. 705,6. |

17. When both factors contain decimals, there must be as many places of decimals in the product as there are decimals in both the factors. According to article 16. Con. III. a unit in the product, before it is pointed, is of the same value of a unit in the factor when the comma is removed. Thus the product of 42,6 by 3, is 1278 before it is pointed, but it does not represent so many whole numbers, but tenths, because when the comma is removed from 42,6, it is in tenths.

If multiplying a number, containing decimals, by a whole number, gives a product, before it is pointed, of the same denomination with the decimal expression in the factor, then by multiplying two numbers together, both having decimals, the product will be of the same denomination, which the decimals of both the factors placed one after the other, would express. The product of 36,48 by 8,16 is 2976768, and the decimals of the factors, written one after the other, would become ten thousandths, consequently the product is in ten thousandths, and must be divided by 10,000, which cuts off the four right hand figures, and renders it 297,6768.

*Pup.* From what you have said, the reason of pointing appears very plain, and I think I shall be able to answer any questions in multiplication of decimals.

*Tut.* To multiply decimals, or whole numbers accompanied by decimals;—

**Place the factors and multiply as in whole numbers; then point off from the product so many places for decimals as there are decimals in both the factors.**

Multiply 223,86 by 2,500. Ans. 559,65000.
Multiply 8,84960 by 2,618. Ans. 22,2425280.
Multiply 520,3 by 0,417. Ans. 110,424.

18. It is sometimes necessary to put one or more ciphers on the left of the product, in order to give the number of decimals required. If you multiply 0,24 by 0,0004, the product of 24 by 4 will be 96, consisting of but two figures, whereas the product ought to consist of 6 figures, and ciphers must be prefixed, thus, 0,000096.

*Multiply 0,217 by* 0,0431. Ans. 0,0093527.

When decimals are to be multiplied by 10, 100, 1000, &c. it is done *by removing the point* 1, 2, 3, &c. *places towards the right.*

Multiply 0,468 by 10. Ans. 4,68.
Multiply 0,624053 by 1000. Ans. 624,053.

---

19. In division of numbers the magnitude of the quotient does not depend on the magnitude of the divisor and dividend, but on their relative magnitudes. If, then, it be required to divide 48,24 by 12, we should observe that 48,24 amount to 4824 hundredths. (17. Con. III.) Now as the divisor and dividend must be of the same denomination, 12 must be reduced to hundredths by annexing ciphers. Thus to divide 48,24 by 12, the numbers will stand as follows,

```
1200)4824(4
     4800
     ----
       24
```

Hence to divide any number accompanied by decimals, by another number having decimals;—

**Equalize the decimal places of the two numbers, viz. if the decimal places in the dividend exceed those in the divisor, annex ciphers to the divisor, till its decimal places are equal to those of the dividend; and if those of the divisor exceed those of the dividend, annex ciphers to the dividend till its decimals are equal to those of the divisor. Then proceed exactly as with whole numbers.**

Divide 5673,21 by 23,0. Ans. 246,660 &c.
Divide 27845,96 by 9,8732. Ans. 2820,3581 &c.
Divide 200,5 by 231. Ans. 0,0867 &c.

---

Vulgar fractions may be reduced to decimal fractions *by annexing ciphers to the numerator, and dividing it by the denominator.*

20. This rule is founded on the following reasons;—The numerator being converted into decimal parts by annexing ciphers, can be divided by the denominator, and by this means becomes a decimal; because the decimal obtained, must bear the same proportion to its denominator that the numerator of the vulgar fraction bears to that denominator. If we wish to reduce $\frac{1}{2}$ to a decimal, we place a cipher at the right of the numerator, and divide it by the denominator, which gives 5 for a quotient, which is 0,5 and has the same proportion to 10, that 1 has to 2. Again, if we reduce $\frac{6}{12}$ to a decimal, we obtain 0,5; and 5 has the same proportion to 10 that 6 has to 12, and the same of any fraction whatever.

| | |
|---|---|
| Reduce $\frac{3}{4}$ to a decimal. | Ans. 0,75. |
| Reduce $\frac{4}{5}$ to a decimal. | Ans. 0,8. |
| Reduce $\frac{7}{8}$ to a decimal. | Ans. 0,875. |

All operations with federal money are according to the rules of decimals. In writing any sum in federal money first write the dollars, then write the comma, and at the right of it place the cents and mills. The cents and mills are decimals of a dollar; and in all operations with Federal Money, the dollars are considered as whole numbers, and the cents and mills as decimals.

### *Questions.*

What are vulgar fractions?

How does it appear that writing the remainder after division, over the divisor, expresses the true value of the remainder?

What are the two terms of the fraction called?

Why are they called numerator and denominator?

How do you reduce fractions to their least terms?

Give the reasons for proceeding as you do for obtaining a common divisor?

How do you multiply fractions together?

Why does multiplying the numerator increase the fraction, and multiplying the denominator decrease it?

How do you divide one fraction by another?

How are whole numbers expressed fractionwise?

How do you find a common denominator?

How does it appear that this will give a common denominator?

What do you do when there are mixed numbers?

What do you do when there are compound fractions?

How are fractions added?

How are they subtracted?

What are decimal fractions?

How are they written?

How do they differ from vulgar fractions?

How do you know what the denominator of a decimal fraction is, and why is it not expressed?

How do ciphers, placed at the right of decimals, affect them?

How do you distinguish decimals from whole numbers?

How do you point the figures in addition and subtraction of decimals?

How do you point them in multiplication and division of decimals?

How does it appear that this will give a true expression?

How do you reduce a vulgar to a decimal fraction?

How does it appear that this will give the true value of the vulgar fraction?

In what respect is Federal Money like decimals?

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## *Examples for Practice.*

1. What is the greatest common divisor of 720 and 1736? Ans. 8.

2. Multiply $\frac{1}{2}$ of 7 by $\frac{3}{6}$. Ans. $1\frac{3}{4}$.

3. Multiply $\frac{1}{3}$ of 5 by $\frac{3}{4}$ of $\frac{2}{7}$. Ans. $\frac{5}{14}$.

4. Multiply $\frac{3}{7}$ of $\frac{5}{9}$ by $\frac{4}{5}$ of $\frac{1}{5}$ of $11\frac{3}{7}$. Ans. $\frac{64}{147}$.

5. Divide $\frac{4}{7}$ by $\frac{2}{3}$. Ans. $\frac{6}{7}$.

6. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. Ans. $\frac{2}{3}$.

7. *Divide* $3\frac{1}{6}$ by $9\frac{1}{2}$. Ans. $\frac{1}{3}$.

8. Divide $4204\frac{1}{6}$ by $\frac{7}{8}$ of 112. Ans. $42\frac{529}{588}$.

9. Add $\frac{3}{7}$, $9\frac{1}{5}$ and $\frac{2}{3}$ of $\frac{1}{2}$ together. Ans. $9\frac{101}{105}$.

10. Add $\frac{7}{10}$ of $4\frac{5}{8}$, $\frac{3}{4}$ of $\frac{1}{3}$ and $9\frac{1}{4}$ together. Ans. $12\frac{59}{80}$.

11. From $96\frac{1}{3}$ take $14\frac{3}{7}$. Ans. $81\frac{19}{21}$.

12. From $14\frac{1}{4}$ take $\frac{2}{3}$ of 19. Ans. $1\frac{7}{12}$.

13. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. Ans. $2\frac{1}{12}$.

14. Add 126,042—0,032—6,568 and 16 together. Ans. 148,642.

15. Add 0,00004—2,—14,6 and 12,6666 together. Ans. 29,26664.

16. Add 4,—0,6—140 and 600 together. Ans. 744,6.

17. From 16 take 0,4689. Ans. 15,5311.

18. From 2 take 0,16289. Ans. 1,83711.

19. Multiply 146,003 by 62,16. Ans. 9075,54648.

20. Multiply 2,046 by 0,0004. Ans 0,0008184.

21. Divide 336,04 by 24,6. Ans. 13,66 &c.

22. Divide 689,6 by 42,368. Ans. 16,274 &c.

23. Reduce $\frac{16}{31}$ to a decimal. Ans. 0,51612 &c.

24. Reduce $\frac{1276}{89672}$ to a decimal. Ans. 0,14228 &c.

# CONVERSATION IV.

## COMPOUND ADDITION, COMPOUND SUBTRACTION, AND REDUCTION.

*Tut.* What is Compound Addition?

*Pup* I cannot tell. I should think by the name, that it was the addition of compound numbers; but what kind I cannot tell.

*Tut.* It is the addition of compound numbers, viz. numbers which increase in a different manner from the simple numbers, by its taking a certain number of one denomination to make one of the next greater, and a different number of this greater to make one of the *next greater*.

We have the following rule for performing

## COMPOUND ADDITION.

**Place the numbers so that those of the same denomination may stand under each other; placing those of the highest denomination on the left, and proceeding orderly towards the right till you have written the lowest denomination.**

**Begin with the right hand column, and find the sum as in simple addition, carrying as many to the next higher denomination from this sum, as this sum contains units of the next higher denomination, and proceed in every respect as in simple addition, except in carrying to the higher denomination, and keeping each denomination separate from the rest.**

Before performing on compound numbers, you must learn the tables of weights and measures, which I will give you with an example annexed to each.

---

### TROY WEIGHT.

| | | | |
|---|---|---|---|
| 24 grains | make 1 penny-weight, | marked | grs. pwt. |
| 20 penny-weights | 1 ounce, | | oz. |
| 12 ounces | 1 pound, | | lb. |

By this weight are weighed jewels, gold, silver, corn, bread and liquors.

| lb. | oz. | pwt. | gr. |
|---|---|---|---|
| 1 | 6 | 15 | 14 |
| 5 | 8 | 6 | 22 |
| 4 | 5 | 13 | 9 |
| 11 | 8 | 15 | 21 |

In this example I begin with the right hand column, and find its sum is 45, which is 45 grains, and because 24 grains make 1 penny-weight, I seek how many times 24 is contained in 45, and find it is once and 21 over; I set down 21, and carry 1 to the next higher denomination, which is pennyweights. I then find the amount of the *column of* pennyweights, and seek how many units of the *next higher* denomination are contained in it, and find it is once and 15 over. I set down 15 and carry 1 to the next column, and proceed in this manner with all the other parts. Numbers in all the other weights and measures are added in the same manner, observing to carry, in all cases, from the lower to the next higher denomination, according to the quantum, which it takes of the lower to make one of the next higher.

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## AVOIRDUPOIS WEIGHT.

| | | | | |
|---|---|---|---|---|
| 16 drams | make | 1 ounce, | marked | dr. oz. |
| 16 oz. | - - | 1 pound, | - - | lb. |
| 28 lb. | - - | 1 quarter, | - - | qr. |
| 4 qr. | - - - | 1 hundred weight, | | cwt. |
| 20 cwt. | - - | 1 ton. | - - | T. |

By this weight are weighed all things of a coarse or drossy nature; such as butter, cheese, flesh, grocery wares, and all metals except gold and silver.

| T. | cwt. | qr. | lb. | oz. | dr. |
|---|---|---|---|---|---|
| 2 | 16 | 2 | 24 | 12 | 11 |
| 6 | 14 | 3 | 16 | 9 | 6 |
| 4 | 18 | 1 | 27 | 15 | 14 |
| 3 | 9 | 2 | 6 | 7 | 8 |

◆

## DRY MEASURE.

| | | | | |
|---|---|---|---|---|
| 2 pints | make | 1 quart, | marked | pts. qt. |
| 2 quarts | - | 1 pottle, | - | pot. |
| 2 pottles | - | 1 gallon, | - | gal. |
| 2 gallons | - | 1 peck, | - | pe. |
| 4 pecks | - | 1 bushel, | - | bu. |
| 2 bushels | - | 1 strike, | - | str. |
| 8 bushels | - | 1 quarter, | - | qr. |
| 5 quarters | - | 1 wey *or* load, | - | wey. |
| 4 bushels | - | 1 coom, | - | co. |
| 2 cooms | - | a seam *or* quarter, | | |
| 6 seams | - | 1 wey, | | |
| 1⅔ weys | - | 1 last, | - - | L. |

The diameter of a Winchester bushel is 18½ inches, and its depth 8 inches. One gallon, dry measure, contains 268⅘ cubic inches.

By this measure are measured salt, lead, ore, oysters, corn and other dry goods.

| bu. | pe. | gal. | pot. | qts. | pts. |
|---|---|---|---|---|---|
| 1 | 2 | 1 | 1 | 1 | 1 |
| 1 | 3 | 0 | 0 | 1 | 1 |
| 0 | 2 | 1 | 1 | 0 | 0 |

## ALE AND BEER MEASURE.

| | | | | |
|---|---|---|---|---|
| 2 pints | make | 1 quart, | marked | pts. qts. |
| 4 quarts | - | 1 gallon, | - | gal. |
| 8 gallons | - | 1 firkin of ale, | - | fir. |
| 9 gallons | - | 1 firkin of beer, | | fir. |
| 2 firkins | - | 1 kilderkin, | - | kil. |
| 2 kilderkins | - | 1 barrel, | - | bar. |
| 3 kilderkins | - | 1 hogshead, | - | hhd. |
| 3 barrels | - | 1 butt, | - - | butt. |

The ale gallon contains 282 cubic inches. In London the ale firkin contains 8 gallons, and the beer firkin 9; the other measures being in the same proportion.

| butt. | bar. | kil. | fir. | gal. | qts. |
|---|---|---|---|---|---|
| 6 | 2 | 1 | 1 | 6 | 3 |
| 5 | 1 | 1 | 0 | 5 | 2 |
| 9 | 2 | 0 | 1 | 7 | 3 |

## WINE MEASURE.

| | | | | |
|---|---|---|---|---|
| 2 pints | make | 1 quart, | marked | pts. qts. |
| 4 quarts | - | 1 gallon, | - | gal. |
| 42 gallons | - | 1 tierce, | - | tier. |
| 63 gallons | - | 1 hogshead, | - | hhd. |
| 84 gallons | - | 1 puncheon, | - | pun. |
| 2 hogsheads | - | 1 pipe *or* butt, | | p. *or* b. |
| *2 pipes* | - | 1 tun, | - - | T. |
| *18 gallons* | - | 1 runlet, | - | run. |
| *31½ gallons* | - | 1 barrel, | - | bar. |

By this measure, brandy, spirits, perry, cider, mead, vinegar and oil are measured.

231 cubic inches make a gallon, and 10 gallons make an anchor.

| T. | p. | hhd. | gal. | qts. |
|---|---|---|---|---|
| 16 | 1 | 1 | 60 | 3 |
| 4 | 0 | 0 | 62 | 2 |
| 6 | 1 | 1 | 14 | 3 |

## CLOTH MEASURE.

| | | | | |
|---|---|---|---|---|
| 2¼ inches | make | 1 nail, | marked | nls. |
| 4 nails | - | 1 quarter, | - | qrs. |
| 4 quarters | - | 1 yard, | - | yds. |
| 3 quarters | - | 1 Ell Flemish, | | Ell Fl. |
| 5 quarters | - | 1 Ell English, | | Ell Eng. |
| 6 quarters | - | 1 Ell French, | | Ell Fr. |

| yds. | qrs. | nls. |
|---|---|---|
| 16 | 3 | 2 |
| 18 | 2 | 3 |
| 9 | 1 | 2 |

| Ell Eng. | qrs. | nls. |
|---|---|---|
| 12 | 4 | 3 |
| 16 | 3 | 2 |
| 14 | 2 | 1 |

## LONG MEASURE.

| | | | | |
|---|---|---|---|---|
| 3 barley corns | make | 1 inch, | marked | bar. c. in. |
| 12 inches | - | 1 foot, | - | ft. |
| 3 feet | - | 1 yard, | - | yd. |
| 5½ yards | - | 1 pole, | - | pol. |
| 40 poles | - | 1 furlong, | - | fur. |
| 8 furlongs | - | 1 mile, | - | mls. |
| 3 miles | - | 1 league, | - | L. |
| 60 geographical miles, *or*<br>69½ statute miles, | | 1 degree, nearly, | | deg. *or* ° |

360 degrees the circumference of the whole earth.

| deg. | mls. | fur. | pol. | yds. | ft. | in. | bc. |
|---|---|---|---|---|---|---|---|
| 40 | 48 | 6 | 32 | 4 | 2 | 10 | 2 |
| 120 | 59 | 7 | 26 | 3 | 1 | 9 | 1 |
| 14 | 11 | 4 | 16 | 1 | 0 | 0 | 2 |

## TIME.

| | | | | |
|---|---|---|---|---|
| 60 seconds | make | 1 minute, | marked | s. *or* ″. m. *or* ′. |
| 60 minutes | - | 1 hour, | - | h. *or* °. |
| 24 hours | - | 1 day, | - - - | d. |
| 7 days - | - | 1 week, | - | w. |
| 4 weeks | - | 1 month, | - | m. |
| 13 months, 1 day and 6 hours, *or*<br>365 days, 6 hours, | | 1 Julian year, | | Y. |

12 calendar months make a year.

| w. | d. | h. | m. | s. |
|---|---|---|---|---|
| 3 | 6 | 22 | 24 | 36 |
| 2 | 4 | 16 | 46 | 58 |
| 1 | 5 | 18 | 52 | 50 |

The following rules are occasionally useful, but not so much used as those already given.

## APOTHECARIES' WEIGHT.

| | | | | |
|---|---|---|---|---|
| 20 grains | make | 1 scruple, | marked | gr. sc. |
| 3 scruples | - | 1 dram, | - | dr. *or* ʒ. |
| 8 drams | - | 1 ounce, | - | oz. *or* ℥. |
| 12 ounces | - | 1 pound, | - | lb. |

Apothecaries use this weight in compounding their medicines ; but they buy and sell their drugs by Avoirdupois weight. Apothecaries is the same as Troy weight, having only some different divisions.

## LAND OR SQUARE MEASURE.

| | | |
|---|---|---|
| 144 inches - - | make | 1 square foot. |
| 9 feet - - - - | - | 1 yard. |
| 30¼ yards, *or* 272¼ feet | - | 1 pole. |
| 40 poles - - - - | - | 1 rood. |
| 4 roods, *or* 160 rods, *or* 4840 yds, | | 1 acre. |
| 640 acres - - - | | 1 mile. |

## SOLID MEASURE.

By solid measure is meant, the measure used to measure bodies that have length, breadth, and thickness.

1728 inches - - make 1 foot.
27 feet - - - 1 yard.
40 feet of round timber, *or* 50 feet of hewn timber, } - a ton *or* load.
128 solid feet, i. e. 8 feet long, 4 feet high, and 4 feet wide, } a cord.

## MOTION.

60 seconds - make a minute, marked ″. ′.
60 minutes - - 1 degree, °.
30 degrees - - 1 sign, - s.
12 signs, *or* 360 degrees, the great circle of the Zodiack.

### *Measures and Numbers not included in the Tables.*

12 particular things make a dozen.
12 dozen make a gross.
12 gross one great gross.
20 particular things 1 score.

6 points - - make 1 line.
12 lines - - - 1 inch.
4 inches - - - 1 hand.
5 feet - - - 1 geometrical pace.
66 feet, *or* 4 poles, - a Gunter's chain.

A hand is used to measure the height of horses.

A quintal of fish is cwt. Avoirdupois.

# COMPOUND SUBTRACTION.

The subtraction of compound numbers is the same as simple numbers, except borrowing and carrying.

*Pup.* From what you have taught me concerning Compound Addition, I expect I must borrow and carry, according to the increase of numbers from one denomination to another.

*Tut.* You must. The following is the rule for Compound Subtraction.

Write the less number under the greater, so that its several denominations may stand under the corresponding denominations of the greater.

Begin with the lowest denomination mentioned, and subtract the lower from the upper number, setting the difference beneath. If the upper number is not so large as the number under it, borrow as many as it takes of this denomination to make one of the next greater, and add to it; then subtract, carrying one to the lower number of the next higher denomination for the one which you borrowed. In every other respect it is like simple subtraction.

| lb. | oz. | pwt. | grs. |
|---|---|---|---|
| 28 | 10 | 18 | 22 |
| 24 | 6 | 12 | 16 |

| T. | cwt. | qr. | lb. | oz. | dr. |
|---|---|---|---|---|---|
| 18 | 16 | 3 | 24 | 12 | 14 |
| 12 | 18 | 2 | 26 | 14 | 12 |

| bu. | pe. | qts. |
|---|---|---|
| 40 | 3 | 2 |
| 16 | 2 | 3 |

| butt. | kil. | fir. | gal. | qts. | pts. |
|---|---|---|---|---|---|
| 68 | 1 | 1 | 6 | 3 | 1 |
| 26 | 2 | 0 | 7 | 2 | 0 |

| deg. | mls. | fur. | pol. | yds. | ft. | in. | b.c. |
|---|---|---|---|---|---|---|---|
| 120 | 58 | 6 | 36 | 4 | 2 | 9 | 1 |
| 86 | 59 | 7 | 38 | 3 | 1 | 10 | 2 |

| T. | p. | hhd. | gal. | qts. |
|---|---|---|---|---|
| 48 | 0 | 0 | 24 | 2 |
| 36 | 1 | 1 | 38 | 3 |

| yds. | qr. | nls. |
|---|---|---|
| 8 | 1 | 2 |
| 5 | 3 | 3 |

| w. | d. | h. | m. | s. |
|---|---|---|---|---|
| 3 | 5 | 20 | 50 | 40 |
| 1 | 6 | 22 | 13 | 58 |

| w. | d. | h. | m. | s. |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
|  | 3 | 8 | 7 | 6 |

*Pup.* These rules are very plain, because they are so much like the simple rules. But it must have been much easier, I think, if these weights, measures, &c had all increased in a decimal ratio, as the simple numbers do.

*Tut.* It would be much easier, and all the rules of arithmetic, in which these weights, measures, &c. are concerned, would be much simplified, if they increased in a decimal ratio. The inconvenience of the present division of weights and measures has been long felt, and the French, in the latter part of the last century, altered all their weights, measures, time, &c. and reduced them to a decimal ratio. This division of weights and measures still continues in France, and will probably soon extend itself to all civilized countries. There can be no serious objection to the change, but a little confusion which it might cause. This would be but trifling compared with the advantages which would result from it. The division of time was abolished a few years after its adoption, and will probably never receive any very essential alteration from its present division.

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Although these numbers do not increase in a decimal ratio, yet they can be reduced to the same denomination, and then the operations with them may be performed as upon decimal numbers. If you have any operation to perform upon 2 lb. 6 oz. 16 pwt. 12 gr. the denominations may be reduced to grains, which is done as follows.

```
   2  6  16  12
  12
 ----
  24
   6
 ----
  30
  20
 -----
  600
   16
 -----
  616
   24
 -----
 2464
1232
   12
 -----
14796
```

As 12 oz. make 1 pound, if you multiply pounds by 12 the product will be in ounces. In the above example there are 2 pounds, which, multiplied by 12, gives 24, the number of ounces in 2 pounds, and as there are 6 oz. beside the pounds, add them to 24, which make 30, the whole number of ounces in 2 lb. 6 oz. Now as 20 pwt. make an ounce, if you multiply the ounces by 20, the product will be in pennyweights, to which add the given pennyweights, and you have the number of pennyweights there are in 2 lb. 6 oz. 16 pwt. Multiply this sum by 24, the number of grains it takes to make a pennyweight, and you have a product in grains, to which add the given grains, and you have the number of grains there are in 2 lb. 6 oz. 16 pwt. 12 gr.

After you have reduced a number of different denominations to one denomination, you may wish to reduce it back again to its original expression. To do this, you must proceed directly the reverse from what you do to reduce it to one denomination. To reduce 14796 grains to their highest denominations, you will proceed as follows.

```
24)14796(616        20)616(30
   144                 60
   ---                 --
    39                 16
    24
    --
    156
    144        12)30
    ---        ---
     12          2 lb. 6 oz. 16 pwt. 12 gr.
```

Here you have grains to reduce to higher denominations, and as 24 grains make 1 pennyweight, dividing the grains by 24, gives the number of pennyweights contained in them. In 14796 grains there are 616 pwt. and 12 grs. over. Pennyweights divided by 20, give ounces, because 20 pwt. make 1 oz. In 616 pwt. there are 30 oz. and 16 pwt. over. Ounces divided by 12, give pounds, because 12 ounces make 1 pound. In 30 oz. there are 2 pounds, and 6 ounces over, so that you have the original number as first given, viz. 2 lb. 6 oz. 16 pwt. 12 grs.

Reducing numbers from a higher to a lower denomination *is called reduction descending*, because the value of the figures, taken separately, descends. When numbers are reduced from a lower to a higher denomination, it is called *reduction ascending*, because the value of the figures, taken separately, ascends or increases. The first example given is in reduction descending, and the second is in reduction ascending.

The examples and what I have said, will be sufficient to convince you of the reason of the following rules for

# REDUCTION.

To reduce compound numbers from higher to lower denominations :

Multiply the highest denomination by such a number as a unit of that denomination will make of the next lower denomination, and to the product add the number given, belonging to the next lower denomination. Proceed in this manner with all the denominations, and the last sum will be the number required.

Reduce 59 lb. 13 pwt. 5 grs. to grains. Ans. 340157.

Reduce 40 cwt. 3 qrs. 24 lb. 12 oz. 14 dr. to drams.
Ans. 1174734.

Reduce 365 d. 5 h. 48′ 48″ to seconds. Ans. 31556928.

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To reduce numbers from a lower to a higher denomination ;—

Divide the given number by such a number as it takes of this lower denomination to make one of the next higher denomination, and the quotient will be the number of the next higher. Then divide this higher by as many as it takes of it to make one of the next higher denomination, and proceed in this manner with the whole ; when the last quotient, with the several remainders, will be the answer.

Reduce 801213 grains to pounds, &c.
Ans. 1390 lb. 11 oz. 18 pwt. 19 gr.

Reduce 9758 pints of brandy to pipes, &c.
Ans. 9 p. 1 hhd. 22 gal. 3 qts.

Reduce 89763 square yards to acres.
Ans. 18 a. 2 r. 7 p. 101 ft. 36 in.

It is sometimes convenient to reduce compound numbers to a vulgar fraction. This is done by reducing the given number to the lowest denomination mentioned, for a numerator, and placing that number which it takes of this denomination to make one of the next higher, for a denominator.

In the example before given, 2 lb. 6 oz. 16 pwt. 12 gr. if you wish to reduce it to a vulgar fraction, reduce it to grains, which, placed for a numerator over 24, will form the fraction $\frac{14796}{24}$. If you wish to reduce this to the fraction of a pennyweight, multiply the denominator by 20, which gives $\frac{14796}{480}$; this is dividing the fraction by 20. (8. Con. III.) You may further reduce it by dividing the fraction by the number which it takes of the denomination which the fraction is in, to make one of the next higher denomination. This rule is not much needed, except to render the nature of reducing a number to a decimal more easy.

Reduce 2 fur. 26 pol. 3 yds. 2 ft. to the fraction of a mile. Ans. $\frac{4400}{5280}$, or $\frac{5}{6}$.

Reduce 6 fur. 16 pol. to the fraction of a mile.
Ans. $\frac{246}{320}$, or $\frac{4}{5}$.

*Pup.* These numbers are now expressed by vulgar fractions, and can be reduced to decimals; is it not much better to reduce them to decimals than it is to have them expressed in vulgar fractions?

*Tut.* It is better, and there is a shorter way to do it than to reduce them to vulgar fractions as taught above.

If for instance you wish to reduce 6 oz. 16 pwt. 12 grs. to decimals, you must consider each of these denominations to have a denominator, viz. $\frac{6}{12}$, $\frac{16}{20}$, $\frac{12}{24}$. You have been taught (20. Con. III.) how to reduce a vulgar to a decimal fraction. In the expression $\frac{6}{12}$, $\frac{16}{20}$, $\frac{12}{24}$, $\frac{12}{24}$ represent a fraction of a pennyweight, and if you reduce $\frac{12}{24}$ to a decimal, you obtain 0,5 which is 5 tenths of a pwt. so that you may now consider the expression to be 6 oz. 16,5 pwt. Now to reduce $\frac{16}{20}$ to a decimal, you will di*vide 16 by 20*, which will give 0,8; but as 16 has become 16,5 the decimal will be 0,825. $\frac{6}{12}$ reduced to a decimal is 0,5 ; but 6 has become 6,825 ; and this divided by 12, gives 0,56875.

Hence, to reduce numbers of different denominations, as of weights, measures, &c. to decimals, we have the following rule.

Write the given numbers perpendicularly under each other for dividends, proceeding orderly from the least to the greatest.

Opposite to each dividend, on the left hand, place such a number for a divisor as will bring it to the next superior denomination, and draw a line perpendicularly between them. Begin with the upper number, and write the quotient of each division, as decimal parts, on the right hand of the dividend next below it ; and so on, till they are all used, and the last quotient will be the decimal sought.

The example which has been given will stand as follows ;—

| | |
|---|---|
| 24 | 12 |
| 20 | 16,5 |
| 12 | 6,825 |
| | 0,56875 |

It is sometimes necessary to reduce a decimal back to the terms of an integer, for which we have the following rule.

Multiply the decimal by that number which it takes of the next less denomination to make one of that denomination in which the decimal is given ; cut off from the right of the product, as many figures as there are figures in the given decimal. Then multiply those figures which you cut off, by that number which it takes of the next less denomination to make one of this, and again cut off, proceeding in this manner till you have multiplied by all the denominations below that in which the decimal is given ; then the number standing on the left will be the answer.

What is the value of 0,629 of cwt. ?

,629
4

2,516
28

4128
1032

14,448
16

2688
448

7,168
16

1008
168

Ans. 2 gr. 14 lb. 7 oz. 2 dr.

2,688

In the decimal 0,629 you have a decimal expression of . hundred weight. Now a unit in this decimal is one housandth of a hundred weight, consequently, if you ultiply this by 4, you obtain a number 4 times as great s this number is, a unit of which will be one quarter of ne thousandth of a hundred weight. In this example he product of ,629 by 4 is 2516 ; which is $\frac{2516}{1000}$ of qr. of wt which, reduced to a mixed number, is 2$\frac{516}{1000}$ qr. so hat you now have 0,1516 of a quarter to reduce to the rms of an integer ; this, multiplied by 28, the number f pounds it takes to make a quarter, gives 14448. which $\frac{14448}{1000}$ of $\frac{1}{28}$ of $\frac{1}{4}$ of a hundred weight; and $\frac{14448}{1000}$, reuced to a mixed number, is 14$\frac{448}{1000}$ lb. 0,448 must now e reduced in the manner the preceding have been, and e same for reducing any decimal to the terms of an inger.

What is the value of 0,397 of a yard ? Ans. 1 qr. 2 nls.

*What is* the value of 0,8593 of a pound Troy ?
Ans. 10 oz. 6 pwt. 5 grs.

Reduce 12 pwt. 14 grs. to the decimal of an ounce.
Ans. 0,6291.

Reduce 3 qrs. 2 nails to the decimal of a yard.
Ans. 0,875.

Previous to the adoption of the Federal Currency, there existed various currencies in the United States, which exist in some places to the present day. As you may have occasion to transact business in those places, I will give you rules by which you may reduce these currencies to Federal Currency. It will be necessary for you to learn the following tables.

## LAWFUL MONEY.

| | | | | |
|---|---|---|---|---|
| 4 farthings | make | 1 penny, | marked. | d. |
| 12 pence | —— | 1 shilling, | —— | s. |
| 20 shillings | —— | 1 pound, | —— | £. |

## FEDERAL CURRENCY.

| | | | | |
|---|---|---|---|---|
| 10 mills | make | 1 cent, | marked | ct. |
| 10 cents | —— | 1 dime, | —— | dim. |
| 10 dimes | —— | 1 dollar, | —— | dol. |
| 10 dollars | —— | 1 eagle, | —— | E. |

The first denomination of Federal Money is an *imaginary coin*, there being none of that kind made. The second, viz. cents, is a copper coin, dimes and dollars are silver, and eagles are gold.

The following is the rule for reducing New England and Virginia; New York, North Carolina and Ohio currencies to Federal Money.*

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* The currencies of N. England and Virginia are the same, and will be called *New England Currency.* The currencies of New York, North Carolina, and Ohio are the same, and will be called *New York Currency*; those of New Jersey, Pennsylvania, Delaware, and Maryland are the same, and will be called *Pennsylvania Currency.*

Write the pounds, and at their right hand write half the greatest even number of shillings. Then find the number of farthings in the given pence and farthings, and if it exceed 12, increase it by 1, or if it exceed 36, increase it by 2, which sum place at the right of half the greatest even number of shillings, and if the shillings be an odd number, increase the left hand figure of the farthings, or the figure next to shillings, by 5. Multiply this number, thus produced, by 10, and divide the product by 3 if it be New-England Currency, and by 4 if it be New-York Currency; cut off the three right hand figures of the quotient, which will be cents and mills; the rest will be dollars.

If there be no shillings, or only 1 shilling, in the given sum, put a cipher in the place of half the number of shillings; then proceed as before.

If pounds only are given to be reduced, multiply by 10, and divide as before, the quotient will be dollars. If there be a remainder, annex three ciphers and divide as before, the quotient will be cents and mills.

When pounds and an even number of shillings are given, annex half the even number of shillings to the pounds, as before, then divide, and the quotient will be dollars; if there be a remainder, ciphers must be annexed, and the quotient will be cents and mills.

Reduce £. 763 N. E. and N. Y. Currencies to Federal Currency.

| | |
|---|---|
| 3)7630 | 4)7630 |
| $ 2543,333 N. E. | $ 1907,50 N. Y. |

Reduce £. 50 7s. 8d. to Federal Money.

| | |
|---|---|
| 3)503830 | 4)50383 |
| $ 167,943 N. E. | $ 125,957 N. Y. |

Reduce £. 10 4d. 2qr. to Federal Money.
Ans. $ 33,396 N. E. $ 25,047 N. Y.

The reasons of this rule are as follows; 1 shilling is $\frac{1}{20}$ of a pound, and if 10 shillings had made a pound, 1 shilling would have been an exact decimal of a pound. But as 20 shillings make a pound, 2 shillings will be an exact decimal of a pound. Wherefore the reason of writing half the even number of shillings.

Again, 960 farthings make a pound. Now if this number be increased by $\frac{1}{24}$ part of itself, it will be an exact decimal of a pound, or 0,001 of a pound; therefore when the farthings are more than 12, it is more than one half of another farthing, or another 0,001 of a pound, and must be increased by 1; and if the farthings are more than 36, they are more than $1\frac{1}{2}$ of 0,001 of a pound, and must be increased by 2. Again, 1 pound, being 20 shillings, is 3 dollars and one third of a dollar. Now it is evident that if you multiply pounds by $3\frac{1}{3}$, the product will be in dollars; and as $3\frac{1}{3} \times 3 = 10$, if you multiply any number by 10 and divide the product by 3, the quotient will be the same as though you multiplied by $3\frac{1}{3}$, and the work is easier.

In reducing N. Y. Currency to Federal Currency, the given sum is prepared in the same manner as before: The reason of multiplying by 10 and dividing by 4, is this; £. 1 N. Y. Currency is $2,50; therefore if you multiply by $2\frac{1}{2}$, you get the answer in dollars, cents and mills; and as 4 is contained in 10, $2\frac{1}{2}$ times, if you multiply by 10 and divide by 4, you get the same number as by multiplying by $2\frac{1}{2}$.

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### To reduce N. Jersey, Pennsylvania, Delaware, and Maryland Currencies to Federal Money.

Reduce the given number to pence, annex a cipher, divide by 9, and add the quotient to the pence. From the sum point off 3 figures, which will be cents and mills; those to the left hand will be dollars.

If there are farthings in the given sum, annex, in the place of the cipher, 2 for 1 farthing, 5 for 2 farthings, 7 for 3 farthings.

If the given sum be pounds only, multiply by 8, annex 3 ciphers to the product, and divide by three; the quotient will be the answer in mills.

Reduce £. 86 6s. 5d. 1qr. to Federal Money.

```
          86  6  5  1
          20
        ----
        1726
          12
     -------
     9)207172
        23019
     --------
Ans. $230,191
```

Reduce £. 736 to Federal Money.

736
8

3)5888000

Ans. $ 1962,666

Reduce £. 42 12s. 8d. to Federal Money.
Ans. $ 113,688.

Reduce £. 684. 6s. 8d. to Federal Money.
Ans. $ 1824,888 &c.

One dollar in N. Jersey, &c. Currency is 7s 6d. which, reduced to pence, is 90d. ; and as 90 pence make one dollar, it is plain that increasing it one ninth of itself will bring it into cents, as $\frac{1}{9}$ of 90 = 10, which, added to 90, make 100, the number of cents in a dollar, and the same with any other number of pence. The reason for annexing a cipher is, that the answer may be in mills, and a more exact expression obtained.

In reducing pounds only, you multiply by 8 and divide by 3, for the following reasons. 1 pound is equal to $2,666, which is equal to $$2\frac{2}{3}$ ; multiplying by 8 and dividing by 3, is the same as multiplying by $2\frac{2}{3}$, which you will understand from what I have before told you.

*Pup.* These rules appear very plain, and I think I shall have no difficulty in reducing any of the currencies to Federal Money.

*Tut.* The rules which I have given you for reducing the several currencies to Federal Money, will probably be sufficient for your business; but there are other currencies which it may be necessary for you to reduce to Federal Money, and your knowledge of the preceding rules will be sufficient to suggest to you a rule for their reduction.

### *Questions.*

*What* is compound addition ?
*How* does it differ from simple addition ?
*What* is compound subtraction ?

How does it differ from simple subtraction ?

What is reduction ?

How is it useful ?

Of how many kinds is it ?

By which of the simple rules is reduction ascending performed ?

By which is reduction descending performed ?

How do you reduce a compound number to a vulgar fraction ?

How does it appear that this will give a true expression of the compound number ?

How do you reduce a compound number to a decimal ?

Why should this give a true decimal expression ?

How do you reduce a decimal to the terms of an integer ?

Why should this give the true value of the decimal ?

How do you reduce N. E. Currency to Federal money ?

Why do you take half the number of shillings ?

Why do you reduce the pence to farthings, and increase them if they exceed 12 *or* 36 ?

Why do you multiply by 10 and divide by 3 ?

Why do you divide by 4, in the reduction of N. Y. Currency ?

How do you reduce Pennsylvania Currency to Federal money ?

Why do you reduce the given sum to pence ?

Why do you divide by 9, and add the quotient to the dividend ?

When the given number is pounds only, why do you multiply by 8, and divide by 3 ?

---

## *Examples for Practice.*

1. A merchant fitted a vessel for sea with the following cargo; 25 tons of iron; 16 tons, 14 cwt. 2 qrs. steel; 7 cwt. 3 qrs. sugar; 50 tons bar iron; what was the weight of the whole cargo? Ans. 92 T. 2 cwt. 1 qr.

2. What is the difference between 126 d. 12 h. 40′ 02″, and 86 d. 18 h. 48′ 58″? Ans. 39 d. 17 h. 51′ 4″.

3. *In 12 pipes how many pints?* Ans. 12096.

4. How many inches from Newburyport to London, it being 2700 miles? Ans. 171072000.

5. In 190080 inches how many yards? Ans. 5280.

6. How many rods from the sun to the earth, it being 95,000,000 miles? Ans. 304,000,000,000.

7. Reduce 7 cwt. 3 qrs. 17lb. 10 oz. 12 dr. to the decimal of a ton. Ans. 0,39538.

8. Reduce 3 qrs. 3 nls. to the decimal of a yard.
Ans. 0,9375.

9. What is the value of 0,387 of a yard?
Ans. 1 qr. 2 nls.

10. What is the value of 0,4689 of a day?
Ans. 11h. 15′ 11″.

11. Suppose a man to be 32 years old, how many seconds has he lived, allowing 365 d. 6 h. 48′ 48″ to a year?
Ans. 1,009,936,896.

12. How many minutes from the creation to the christian era, it being 4004 years? Ans. 2,104,840,032.

13. How many times does the wheel which is 18 feet, 6 inches in circumference, turn round in the distance of 150 miles? Ans. 42810 times, and 180 inches over.

14. In 5529600 solid inches, how many cords of wood?
Ans 25.

15. How many square feet in a square mile?
Ans. 27878400.

16. Reduce £825, 16s. 4d. N. E. currency to Federal Money. Ans. $2752,723.

17. Reduce £64, 12s. N. Y. currency to Federal Money. Ans $161,50.

18. Reduce £120, 15s. 4d. N. E. and N. Y currencies to Federal Money? Ans. $102,556 N. E.
[$310,917 N. Y.

19. Reduce £126 12s. 6d. N. Y. Penn and N. E. currencies to Federal Money. Ans. $ 316.562 N. Y.
[$ 422,083 N, E. $ 337,66 Penn.

*20.* *Reduce* £150 13s. 4d. Penn. currency to Federal *Money*. Ans. $ 401,777.

# CONVERSATION V.

### COMPOUND MULTIPLICATION, COMPOUND DIVISION, AND DUODECIMALS.

*Tut.* You have been taught in the preceding conversation that compound numbers may be multiplied and divided by each other, by first reducing them to the same denomination, and then proceeding as with simple numbers. But when a compound number is to be multiplied by a simple number, the best method is to multiply the compound number by the simple number, without reducing it to one denomination; for which we have the following rule for

## COMPOUND MULTIPLICATION.

Write the compound number, and under its lowest denomination write the simple number by which it is to be multiplied. Begin with the lowest denomination and multiply it by the multiplier; divide this product by that number which it takes of the number multiplied to make one of the next higher denomination; write the remainder after division under the number multiplied, and add the quotient to the product of the simple number and the next higher denomination, and proceed in this manner with all the denominations to the last, which must be multiplied as a simple number.

Multiply 12 lb. 8 oz. 6 pwt. 16 grs. by 8.

| | | | |
|---|---|---|---|
| 12 | 8 | 6 | 16 |
| | | | 8 |
| 101 | 6 | 13 | 8 |

*Pup.* I do not perfectly understand the reason of dividing the product by that number which it takes to make one of the next higher denomination; I should like to have you explain it.

*Tut.* In order to give you a distinct understanding of the whole work, I will multiply each denomination of the compound number by the simple number, and set their products separately, as follows.

| lb. | oz. | pwt. | grs. |
|---|---|---|---|
| 12 | 8 | 6 | 16 |
| 8 | 8 | 8 | 8 |
| 96 | 64 | 48 | 128 |

Here we have 96lb. 64oz. 48pwt. 128grs; now if we begin with 128grs. and divide by 24, the number of grains it takes to make a penny-weight, we get the number of penny weights there are in 128 grains, which is 5, and 8 remainder; if we set down the remainder and add these 5pwt. to 48pwt. we shall have 96 lb. 64 oz. 53 pwt. and 8 grains. If we divide 53 pwt by 20, the number of penny-weights it takes to make an ounce, we shall get the number of ounces there are in 53 pwt. which is 2 and 13 remainder; if we add this quotient 2, to 64, we shall have the number of ounces there are in 64 oz. 48 pwt. 128 grs. viz. 66 oz. If we now divide 66 oz. by 12, the number of ounces it takes to make a pound, the quotient will be the number of pounds contained in 66 oz. viz. 5 lb. and 6 oz. over; add 5 lb to 96 lb and there will be 101 lb. 6 oz. 13 pwt. 8 grs. the same as by the first process.

*Pup.* The reason, then, of multiplying and dividing as we do, is the same as for carrying for ten in simple multiplication.

*Tut.* It is the same; the only difference being, that the numbers do not increase in a uniform ratio. and require that we carry, from one denomination to another, such a number as it takes of the former to make one of the latter.

Multiply 16 cwt. 2 qr. 24 lb. by 6.
Ans. 100 cwt. 1 qr. 4 lb.

Multiply 125 d. 10 h. 14′ 42″ by 9.
Ans. 1128 d. 20 h. 12′ 18″.

Multiply 12 lb. 9 oz. 13 pwt. 23 grs. by 7.
Ans. 89 lb. 7 oz. 16 pwt. 17 grs.

*What is the weight of 8 hhds. sugar, each weighing 4 cwt. 2 qr. 14 lb.?* Ans. 37 cwt.

When it is required to multiply a number, exceeding 12 ; resolve the number into two or more factors ; then multiply first by one of them, and then their product by one of the others, and so on with all the factors, and the last product will be the answer.

Multiply 12 lb. 6 oz. 5 pwt. by 16.

| | | |
|---|---|---|
| 12 | 6 | 5 |
| | | 8 |
| 100 | 2 | 00 |
| | | 2 |
| 200 | 4 | 00 |

*Pup.* Will this give the same answer as it would to multiply by 16 ?

*Tut.* The answer will be the same ; for multiplying by 8 gives 8 times the multiplicand, and then multiplying by 2 doubles this product, which is the product by 16, as twice 8 is 16.

*Pup.* I now understand it, and see that it must be much easier than to multiply by 16.

*Tut.* You may multiply the following numbers together.

Multiply 14 T. 12 cwt. 2 qrs. 16 lb. by 36.
Ans 526 T. 15 cwt. 0 qrs. 16 lb.

Multiply 126 d. 22 h. 48′ 12″ by 56.
Ans. 7110 d. 16 h. 59′ 12″.

Frequently the multiplier cannot be resolved into factors, and express the exact number. When this is the case, take the number nearest to it which can be resolved into factors, and find the product of the multiplicand by this number. Then add or subtract from this product so many times the multiplicand as the number resolved exceeds or falls short of the given number.

Multiply 14 cwt. 2 qr. 18 lb. by 17.

```
 14   2   18
           4
-------------
 58   2   16
           4
-------------
234   2    8
 14   2   18
-------------
249   0   26
```

Here we find that 17 cannot be resolved into factors, but 16 can, 4 times 4 being equal to 16. The product by 16 falls short of the product by 17 by once the multiplicand, consequently we must add the multiplicand to its product by 16, and it gives the product by 17. 18 might have been taken instead of 16, to resolve into factors, which would have given a product too large by once the multiplicand, and the multiplicand must have been subtracted from the product.

Multiply 12 lb. 8 oz. 6 pwt. 18 grs. by 44.
Ans. 558 lb. 6 oz. 17 pwt.

Multiply 2 d. 16 h. 24′ by 37. Ans. 99 d. 8 h. 48′.

---

When the multiplier exceeds 144, first multiply the multiplicand by 10, then that product by 10, which gives the product of the multiplicand by 100, and if the multiplier be even hundreds, multiply this product by the number of hundreds, and the product will be the answer. If there be odd numbers, multiply the product of 10 by the number of tens, and the multiplicand or given number by the number of units; then the sum of these products will be the answer.

Multiply 12 cwt. 2 qr. 16 lb. by 160.

```
  12   2   16
           10
--------------
 126   1   20          126   1   20
           10                     6
--------------         -------------
1264   1    4          758   2    8
 758   2    8
--------------
2022   3   12
```

Here I first find the product of the multiplicand by 10; then I multiply this product by 10, which gives the product by 100; I then find the product of the multiplicand by 60, by multiplying the product by 10, by 6; and the product of the multiplicand by 60, added to the product of the same by 100, is the answer.

What will 16 cwt. 3 qr. 14 lb. of sugar cost at $10,50 per cwt.? Ans. $117,18.

What will 4 cwt. 2 qr. 18 lb. tobacco cost at $6,25 per cwt.? Ans. $29,12.

---

When the numbers are such that the lower denominations can be resolved into even parts of the highest denomination; to obtain the price of the lower denomination, take such parts of the price as the lower denominations are parts of the highest denomination, and these several parts, added to the product of the highest denomination by the price, will be the answer.

What will 12 cwt. 2 qr. 14 lb. sugar cost at $8,25 per cwt.?

```
8 — 2 ) 8,25
          12
       ------
       99,00 price of 12 cwt.
        4,12 price of half cwt. or 2 qr.
        1,03 price of ⅛ of a cwt. or 14 lb.
       ------
Ans. $ 104,15
```

What will the freight of 42 cwt. 3 qrs. 21 lb. be, at $0,42 per cwt.? Ans. $18,03.

What will 8 yds. 3 qrs. 2 nls. cost, at $7,25 per yard? Ans. $64,34.

* * *

Compound numbers may be *divided* by considering each denomination as a distinct dividend; and as we *multiplied* each separately, we will reverse the order of the work, and *divide* each separately. If we take the example given in multiplication, and reverse the order of the work, we shall obtain the former number.

```
8)96    8)64    8)48    8)128
 ——      ——      ——      ——
 12       8       6       16    12lb. 8oz. 6pwt. 16grs.
```

As in multiplication we increased each number 8 times, so in division we decrease each number 8 times.

But as we have, in multiplication, a shorter method of performing the work, so in division we can divide a compound number by a simple number without the trouble of the above process. The following is the general rule for

## COMPOUND DIVISION.

Place the numbers as in simple division; then divide the highest denomination by the divisor, and reduce the remainder to the next lower denomination, adding to the product the given number of that denomination to which it is reduced; then divide this number by the divisor, and reduce the remainder to the next lower denomination, adding the given number of this denomination as before, and proceed in this manner with all the denominations, when the remainder, if there is any, must be expressed by a fraction; then the sum of the several quotients will be the answer.

If we take the product from the example given in multiplication, and divide it by 8, we shall obtain the number there given, and the work will stand as follows,

```
     lb.  oz. pwt. grs.
   8)101   6   13   8
     ———————————————
      12   8    6  16
```

Here we first find how often 8 is contained in 101, and find it is 12 times and 5 over; 5 here means 5 pounds, and must be reduced to ounces, and the given ounces added to them, which make 66 oz.; 66 divided by 8, give a quotient of 8, and 2 remainder; this remainder must now be reduced to penny-weights, and the given penny-weights added, which make 53; 53 divided by 8, give a quotient of 6, and 5 over; 5, reduced to grains, make, with the given grains, 128, which, divided by 8, give 16 for a quotient; thus you have the same number for an answer in this operation, which was given in multiplication, consequently, compound multiplication and division prove each other, the same as simple multiplication and division do.

*Pup.* All this appears very plain, and the similarity of the compound to the simple rules is so great that I think it will require only a little practice to make them perfectly familiar.

*Tut.* The compound rules are not so difficult as they at first appear to be, provided you understand them well; if you have not got the tables well, and do not understand the nature of the rules, they will always trouble you, and you had better not leave them without a full acquaintance with their principles.

Divide 48 lb. 8 oz. 12 pwt. 14 grs. by 9.
Ans. 5 lb. 4 oz. 19 pwt. $4\frac{2}{9}$ grs.

Divide 4 T. 16 cwt. 2 qr. 24 lb. by 6.
Ans. 16 cwt. 0 qr. $13\frac{1}{3}$ lb.

---

When the divisor is so large as to make it inconvenient dividing by it, and it can be produced by the multiplication of two or more numbers, divide first by one of those numbers, and then that quotient by one of the others, and so on with the whole; when the last quotient will be the answer.

Divide 168 cwt. 3 qrs. 14 lb. by 56.

$$\begin{array}{r|rrrr} 8 & 168 & 3 & 14 & \\ \hline 7 & 21 & 0 & 12 & 4 \\ \hline & 3 & 0 & 1 & 12 \end{array}$$

Ans. 3cwt. 0qr. 1lb. 12oz.

Here by dividing 168cwt. 3qrs. 14lb. by 8 we obtain one eighth of this number, and then dividing this quotient by 7, we get one seventh of the quotient, and $\frac{1}{7}$ of $\frac{1}{8}$ is $\frac{1}{56}$.

Divide 1061cwt. 2qrs. by 28. Ans. 37cwt. 3qrs. 18lb.

Divide 156d. 18h. by 36. Ans. 4d. 8h. 50′.

When the divisor cannot be produced by the multipliation of two or more numbers; the division must be erformed by dividing by the whole number at once.

If the quotient which is sought, were known, we might, y adding it to, or subtracting it from the dividend a cerain number of times, and increasing or diminishing the ivisor at the same time by as many units, alter the quesion so that the divisor might be resolved into factors, vhich would give the same quotient, and the analogy beween multiplication and division preserved. But the uotient being unknown before the operation, this canot be done.

Divide 249cwt. 0qr. 26lb. by 17.

```
17)249 0 26(14 2 18
   17
   ---
    79
    68
   ---
    11
     4
   ---
    44
    34
   ---
    10
    28
   ---
    80
   20
    26
  ----
  306
  17
  ----
  136
  136
  ----
```

In this example, I begin and divide 249 as in simple numbers; obtain 14 for a quotient, and have 11 remainder, which I reduce to quarters; divide them by the divisor, and get 2 for a quotient, which I place so as to be kept separate from the 14cwt. The 10 remaining I reduce to pounds, and again divide, and obtain 18 for a quotient.

*Pup.* I understand this, and think it is very plain, there being nothing different in the principles of the rule from what I have been taught before.

Divide 126cwt. 3qrs. 21lb. by 26.
Ans. 4cwt. 3qrs. 14lb.

Divide 26cwt. 2qrs. 14lb. by 13.
Ans. 2cwt. 0qr. 5lb. 6oz. $2\frac{6}{13}$dr.

◆

When the price of several hundred weight is given, and you wish to find the price of one hundred weight; divide the whole price by the number of hundred weight given, and the quotient will be the answer.

If, having the price of several hundred weight given, you wish to find the price of a quarter, a pound, &c.; first find the price of a hundred weight, and then the price of the lower denominations, by dividing this price by those numbers which will reduce a number from a hundred weight to the denomination for which you wish to find a price.

If 8 cwt. sugar cost $48, what is that per cwt.?

```
        8)48
        ----
    Ans. $6
```

If 16 cwt. sugar cost $148, what is that per pound?

```
16)148(9,25 price of cwt.
   144            4)9,25
   ---            ------
     40          28)231(8,2
     32             224
     --             ---
      80             70
      80             56     Ans. $0,082
                     --
                     14
```

If I give $450, for 210 cwt. raisins, what do they cost per cwt.? Ans. $ 2,142.

---

When the given quantity consists of several denominations, as of hundreds, quarters, pounds, &c. and the price of the whole given, to find the price of any particular denomination, first reduce the given quantity to the denomination for which you wish to find a price, and divide the price of the whole by it, when the quotient will be the answer. If the given quantity consist of lower denominations than the one for which you wish to find the price; reduce the given quantity to the lowest denomination mentioned, and find the price for that, and then find the price for the other denominations, by multiplying this price by such numbers as will reduce a number from this denomination to the denomination for which a price is wanted.

If I give $124 for 16 cwt. 2 qr. of sugar, what is that per quarter?

```
16  2
 4
---
66)124(1,878      Ans. $1,878.
    66
    ---
    580
    528
    ---
     520
     462
     ---
      580
      528
      ---
       52
```

By reducing the whole quantity to quarters and dividing the price by that number, you get the price of one quarter; for if 66 quarters cost $124, one quarter will cost *one sixty* sixth part of the whole price. The ciphers *annexed to* the remainders reduce them to cents and *mills, and* the quotients arising from them are the same.

If 126 cwt. 2 qr. 14 lb. 12 oz. cost $482,50, what is it worth per quarter?

126  2  14  12
4
———
506
28
———
4062
1012
———
14182
16
———
85104
14182
———
226924)482,500(2,1262 the price of an ounce.
453848
———
286520
226924
———
595960
453848
———
1421120
1361544
———
595760
453848
———
141912

2,1262
16
———
127572
21262
———
$0,034,0192 price of 1lb.
28
———
2721536
680384
———
$0,952,5376 pr. of 1 qr.

Ans. $0,952 per qr.

Here we find that one ounce is worth 2 mills and 0,1262 of a mill; this price, multiplied by 16, gives the price of one pound, viz. $0,340,192; which, multiplied by 28, gives the price of one quarter, viz. $0,952. The price of one quarter, multiplied by 4, will give the price of one hundred weight, and the price of cwt. multiplied by the number of cwt. will give the price of the given hundreds, which, with the prices of the other given denominations, will be the price of the whole as it was given.

If 12 cwt. 3 qr. 12 lb. cost $601,75 what is 1 qr. worth? Ans. $11,70.

*Pup.* I shall have no difficulty, I think, in performing any questions in these rules, and should like now to attend to Duodecimals, that I may learn what they are, for I know nothing about them, and cannot determine any thing about them by their name.

◆

*Tut.* Duodecimals are a species of compound numbers, consisting of different denominations, but resembling simple numbers in this, that they increase in a uniform ratio, from one denomination to another. The ratio is 12, and it is this which gives the name, *Duodecimals*, to the rule.

Feet are the highest denomination in this ratio, there being no denomination consisting of 12 feet. Feet are divided into 12 inches each, called primes; an inch is divided into 12 seconds; a second into 12 thirds; a third into 12 fourths, and so on to as low denominations as you please, there being no limit to the division. These several denominations are distinguished from each other by the following marks; feet are marked ft.; inches, or primes, thus ( ′ ); seconds ( ″ ); thirds ( ‴ ); fourths ( ⁗ ); and so on with all the denominations. These marks are called the indices of the terms. The following is the rule for

## DUODECIMALS.

Write the multiplicand, and under it write the multiplier, so that feet may stand under feet, inches under inches, &c. and draw a line underneath. Multiply the multiplicand by the several terms of the multiplier, successively, according to the rule for compound multiplication, placing the first term of each of the partial products under its multiplier, and carrying one for every twelve, from the lower to the next higher denomination, through the whole work; then the sum of these several products will be the answer. The *left hand* number will be the feet, the next inches, and so on with *the whole.*

It may be proper here for me to tell you, that feet multiplied by feet, give feet for a product; feet multiplied by inches, give inches; feet multiplied by seconds, give seconds, &c. Inches multiplied by inches, give seconds; inches multiplied by seconds, give thirds; the product of any two terms will be in that denomination, which the sum of their indices represent. If you multiply 2 ft. by 6′, the product will be in inches; if 4′ be multiplied by 6″, the product will be in thirds, &c.

Multiply 4 ft. 2′ 6″ by 2 ft. 6′ 4″.

$$\begin{array}{rrrrr}
 & & 4 & 2' & 6'' \\
 & & 2 & 6 & 4 \\
\cline{3-5}
 & 1 & 4 & 10 & 00 \\
2 & 1 & 3 & 00 & \\
8 & 5 & 0 & & \\
\hline
10 & 7' & 7'' & 10''' & 00''''
\end{array}$$

Here I begin with the right hand figures of both factors, and find their product is 24, and as we carry for 12, I divide 24 by 12, and find there are twice 12, and nothing over; I then multiply the next figure of the multiplicand by 4, and add 2, which were to carry from the other product, to this product. This does not amount to 12, therefore I set down the whole, and multiply the next figure of the multiplicand by 4, and find there is once 12, and 4 over, which I set down, carrying the 1, one place to the left. I then multiply the multiplicand in the same manner, by the next figure of the multiplier, setting the first term of the product directly under the figure of the multiplier by which I multiply. I next multiply by the last figure of the multiplier, the product of which, by the last figure of the multiplicand is feet, because the two terms are feet. I then add the partial products together, keeping each term separate, and carrying for 12 as in multiplying; then the sum of these products is the answer.

*Pup.* I understand the manner of operation, but do not see why that feet multiplied by inches, should give inches any more than feet; or why inches, multiplied by seconds should give thirds for a product any more than that they should give inches or seconds.

*Tut.* You have not considered the nature of these terms I expect, or you would not have doubted the truth of the rule. In duodecimals all the terms below feet are fractions of a foot; thus 1 inch is $\frac{1}{12}$ of a foot; 2 inches are $\frac{2}{12}$ of a foot; and any number of inches may be expressed fractionwise, by writing the given inches for a numerator and 12 for a denominator. Hence if you multiply 6 feet by $6'$ you will have 6 feet to multiply $\frac{6}{12}$ of a foot, and the product of the factors will be $\frac{36}{12}$ of a foot, or 3 feet. (5. Con. III.) Again, if you are to multiply $6'$ by $6''$, you have $\frac{6}{12}$ of a foot to be multiplied by $\frac{6}{12}$ of $\frac{1}{12}$ of a foot; the product of $6'$ by $6''$ is $36'''$, which is $\frac{36}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of a foot, equal to $\frac{36}{1728}$ of a foot, equal to $\frac{1}{48}$ of a foot, (7. Con. III.) 6 inches are $\frac{1}{2}$ of a foot, consequently, if you multiply any number of feet by 6, you obtain a product of one half the value of the feet multiplied, (5. Con. III.) and the same of all the fractional parts of a foot.

*Pup* I now understand why the product of these fractions of a foot should be in a smaller denomination than either of the factors; it is exactly like the multiplication of vulgar fractions.

*Tut.* It is like the multiplication of vulgar fractions having a common denominator; and you ought to have a good understanding of this or you will be very liable to make mistakes.

The manner of operating, as I have directed, is different from the usual mode; but it is more analogous to the manner of operating in simple and compound multiplication, and is equally convenient, and more easily learned than the ordinary way. The usual method is to begin with the highest denomination of the multiplier and multiply the multiplicand by it; then take the next lower term of the multiplier and proceed as with the first, carrying the excess of 12s to the right; and in this manner multiply by each term of the multiplier, when the terms of the product will stand under the corresponding terms of the multiplier. But the advantage of this is not sufficient to compensate for the trouble of learning it. The following example is performed in this manner.

| | | | | |
|---|---|---|---|---|
| 4 | 2′ | 6″ | | |
| 2 | 6 | 4 | | |
| 8 | 5 | 0 | | |
| 2 | 1 | 3 | 00‴ | |
| | 1 | 4 | 10 | 00⁗ |
| 10 | 7′ | 7″ | 10‴ | 00⁗ |

This rule is used to find the dimensions of wainscotting, plastering, painting and paving, and the contents are given in square yards. The following is the manner in which

PAINTERS AND JOINERS

find the dimensions of their work.

Take a line and fasten it to any spot on the wall of the room; then apply the line to the wall, going round the room and keeping the line applied close to the wall, till you arrive at the spot where the line was fastened; then find how many feet and inches the line measures, and call its length the compass, or round, which, multiplied by the height of the room, and the product, divided by 9, will be the contents of the work in yards.

If the compass of a room be 64 ft. 10′, and the height 9 ft. 6′, what are the contents in yards?

| | | | |
|---|---|---|---|
| 64 | 10 | | |
| 9 | 6 | | |
| 62 | 5 | 00 | |
| 583 | 6 | | |
| 9)645 | 11′ | 00″ | |
| 71 | 6 | | Ans. 71 yds. 6 ft. 11′. |

*Pup.* After dividing there are 6 feet over, and I do not see why they could not have been reduced to yards, for 3 feet make a yard.

*Tut.* A yard here does not mean a yard in length, but a square yard, by which is meant so much of the wall as will measure 3 ft. in length, and 3 ft. in breadth. If you take a square piece of the wall, 3 ft. long and 3 ft. broad, it *will contain 9 ft.* and this is the reason why 6 ft. cannot *be expressed in yards.*

*Pup.* I now see the reason of this, and why dividing by 9 gives an answer in yards; it is because 9 square feet make a square yard.

## *Questions.*

What is compound multiplication?

What is the rule for compound multiplication?

When the multiplier exceeds 12, how do you proceed?

When the given multiplier cannot be resolved into exact factors, what do you do?

When the multiplier exceeds 144, how do you proceed?

When the lower denomination can be resolved into even parts of the highest denomination, how do you obtain the price?

What is compound division?

How may compound numbers be divided?

What is the rule for compound division?

When the divisor is so large as to make it inconvenient dividing by it, how do you proceed?

When the divisor cannot be resolved into factors, what is to be done?

When the price of several hundred weight is given, how do you find the price of one hundred weight? Of one quarter? Of one pound, &c.?

When the quantity given consists of several denominations, and the price of the whole given, how do you find the price of any particular denomination?

What are duodecimals?

Why are they called duodecimals?

What is the highest denomination in this rule?

How are the several denominations distinguished from each other?

For what number do you carry?

What is the product of feet by inches? Inches by inches? Inches by seconds? Seconds by thirds, &c.?

What is the ordinary method of performing this rule?

How do painters and joiners find the contents of their work?

## *Examples for Practice.*

1. In 36 hhds. of sugar, each weighing 12 cwt. 2 qr. 1 lb. how many hundreds, quarters, pounds, &c. ?
Ans. 452 cwt. 2 qr. 8 lb.

2. A man has 46 silver cups, each weighing 6 oz. 10 pwt. 9 gr. what is the weight of the whole ?
Ans. 24 lb. 11 oz. 17 pwt. 6 gr.

3. If a man travel 40 miles, 6 furlongs, and 36 rods in one day, how far will he travel in 26 days ?
Ans. 1062 mls. 3 fur. 16 rods.

4. 28 men bought 258 cwt. coffee, which was shared equally among them ; what was each man's share ?
Ans. $9\frac{6}{28}$ cwt. or 9 cwt. 3 qr. 12 lb.

5. There are 248 cwt. 3 qr. of sugar to be divided among 29 men ; what is each man's share ?
Ans. 8 cwt. 2 qr. 8 lb. $11\frac{1}{29}$ oz. *or* $8\frac{16}{29}$ cwt.

6. If a man travel 648 mls. 4 fur. in 26 days, how far would he travel per day ?
Ans. 24 mls. 7 fur. $21\frac{14}{26}$ pls. *or* $24\frac{24}{26}$ mls.

7. In the Lunar circle of 19 yrs. of 365d. 5h. 48′, 48″. how many days, &c. ? Ans. 6939 d. 14h. 27′ 12″.

8. How many square feet in a board 17 ft. 7′ long, and 1 ft. 5′ wide ? Ans. 24 ft. 10′ 11″.

9. What are the contents of a floor, 48 ft. 6′ long, and 24 ft. 3′ broad ? Ans. 1176 ft. 1′ 6″.

10. What are the contents of a ceiling, 43 ft. 3′ long, and 25 ft. 6′ broad ? Ans. 1102 ft. 10′ 6″.

11. The length of a room is 20 ft. its breadth 14 ft. 6′, and its height 10 ft. 4′ ; how many yards of painting are in it, deducting a fire place of 4 ft. by 4 ft. 4′, and two windows, each 6 ft. by 3 ft. 2′ ? Ans. $73\frac{2}{27}$ yds.

12. There is a house which has 12 windows 4 ft. 6′ long, and 2 ft. 10′ wide ; 8 windows 3 ft. 9′ long, and 2 ft. 6′ wide ; 16 windows 4 ft. 2′ long, and 3 ft. 1′ wide ; how many square feet do all the windows contain ?
Ans. 433 ft. 6′ 8″.

13. How many cubic feet* of wood in a load, 6 ft. 7′ long, 3 ft. 5′ high, and 3 ft. 8′ wide? Ans. 82 ft. 5′ 8″ 4‴.

14. There is a stock of 14 boards; each board is 14 ft. 6′ long, and 1 ft 10′ wide; how many feet are there in the whole stock? Ans. 372 ft. 2′.

15. The sun passes through 15 degrees in an hour; two places differ in longitude 31° 37′ 3″; what is the difference, in time, of the sun's coming to the meridian of those places? Ans. 2h. 6′ 3″.

16. How many square yards are there in the wainscotting of a room 18 ft. long, 16 ft. 6′ wide, and 9 ft. 10′ high? Ans. 2920 ft. 6′.

17. Required the solid contents of a wall 53 ft. 6′ long, 12 ft. 3′ high, and 2 ft. thick. Ans. 1310 ft. 9′.

# CONVERSATION VI.

## SIMPLE AND COMPOUND INTEREST.

*Tut.* What is interest?

*Pup.* Interest is what one man pays to another for the use of money.

*Tut.* Very well. Interest is an allowance made by the borrower to the lender. If A. should borrow money of B., he might traffic with that money, obtain gain, and thus be benefited by the use of it. But B. would be deprived of the use of the money, and sustain a loss by having lent it to A. Hence the reason of the borrower's giving interest to the lender for the use of the money borrowed.

The money lent is called the *Principal.*

The interest or allowance made is called the *Rate* per cent.

The principal and interest added together are called the *Amount.*

---

* *A cubic foot is that which is a foot long, a foot wide, and a foot thick.*

Interest is of two kinds, *Simple* and *Compound*. Simple interest is that which is allowed for the principal only, or money lent.

Interest is considered as payable at the close of every year, from the date of the note or bond. When the borrower does not pay the interest, but keeps it in his own hands, it is considered as added to the principal, or money lent, and interest is allowed for it the same as for the money lent. This is called *Compound Interest.*

The rate per cent of interest is different in different places. The medium rate is 6 per cent, or $6 a year for every $100 lent. The following is the

### *General Rule.*

If the interest is for only one year, multiply the principal by the rate, and from the product cut off two more places for decimals, than there are decimals in the multiplicand. The figures at the left of the point will be dollars, and those at the right, cents and mills.

For more than one year, multiply the interest for one year, by the number of years.

For months, take proportional parts of the interest for one year, viz. for 6 months, $\frac{1}{2}$; for 5 months, $\frac{1}{3}$ and $\frac{1}{12}$; for 4 months, $\frac{1}{3}$, &c.

For days, take proportional parts of the interest for one month, calling 30 days a month.

What is the interest of $18,24 for two years and six months, at 6 per cent?

```
    18,24
        6
  2)10944 interest for one year.
        2
    21888 interest for two years.
     5472 interest for six months.
 $ 2,7360 interest for 2 years and 6 months.
```

The reason of multiplying by the rate will appear plain, when you consider that 6, or any other figure expressing the rate, is 0,06 &c. of a dollar; or the rate is so many cents on the dollar for one year. Hence the reason of pointing the product as we do. But as 6 per cent is the more general rate, allowed by law, there is not, perhaps, a better method of casting interest, than by the following *Rule.*

Write the given sum for a multiplicand, and half the even number of months for a multiplier; if there be an odd month, call it 30 days, which add to the given days, if any, and seek how often 6 are contained in their sum, and place the figure for a decimal at the right of half the even number of months. Multiply the principal by these figures, and from the product point off two more figures for decimals than there are decimal figures in both the factors. The figures to the left of the point will be dollars, and those to the right, cents and mills, and parts of a mill, which is the interest required.

When there is a remainder in taking one sixth of the days, reduce it to a vulgar fraction, for which take proportional parts of the multiplicand.

"If the remainder be 1 = $\frac{1}{6}$ divide the multiplicand by 6.
If - - - 2 = $\frac{1}{3}$ - - - - by 3.
If - - - 3 = $\frac{1}{2}$ - - - - by 2.
If - - - 4 = $\frac{2}{3}$ - . - by 3 twice.
If - - - 5 = $\frac{1}{2}$ and $\frac{1}{3}$ - - by 2 and 3."

These quotients must be added to the product of the given sum by half the even number of months, and the sum will be the interest required.

When the days do not amount to 6, place a cipher at the right of half the even number of months, then proceed in all respects as before.

What is the interest of $ 18,48 for 2 years, 7 months and 4 days?*

```
,3—3)18,48
      15,5
     -----
      9240
     9240
    1848
       616
       616
   -------
  $2,876,72
```

---

* When the rate per cent is not mentioned in the obligation, it is always considered the interest established by law, which is called *lawful interest.*

In this example I first seek how many months there are, and find they are 31. I take half the even number months for the following reasons. One dollar at 6 per cent interest, draws 6 cents in one year; and as there are 12 months in a year, the interest is half a cent per month on the dollar, consequently if we multiply dollars by half the number of months, we get the interest in cents. When there are not 2 months, so that the half cannot be expressed by 1, it is evident that it must be done by a decimal expressing ½, which is 0,5, and 5 times 6 are 30. When there are odd days, less than 6, it is evident that their value can be obtained by taking proportional parts of the multiplicand. When the days are less than 6, a cipher is annexed, that the quotient arising from the division of the multiplicand by a fractional part of 6, may stand in its proper place, otherwise it would be increased ten times.

What is the interest of $124,18 for 2 years and 8 months? Ans. $19,868.

What is the interest of $240,16 for 3 years, 5 months and 1 day? Ans. 49,672.

What is the interest of $958,54 for 5 days? Ans. $0,798.

---

When the rate is any other than 6 per cent, first find the interest at 6 per cent, then divide the interest so found by such parts as the interest, at the rate required, exceeds or falls short of the interest at 6 per cent, and the quotient added to or subtracted from the interest at 6 per cent, as the case may be, will give the interest at the rate required.

What is the interest of $246,25 for 2 years and 6 months, at 5 per cent?

246,25
15

123125
24625

6)36,9375
61562

Ans. $30,7813

What is the interest of $ 162,14 for 2 years and 4 months at 7 per cent? Ans. $ 26,482.

What is the interest of $ 64,18 for 1 year and 10 months, at 4 per cent? Ans. $ 4,706.

---

It frequently happens that the debtor wishes to make partial payments of his Bonds, Notes, &c. Interest on obligations of this description, is cast in the following manner, as established by the courts of law in Massachusetts:

> "Cast up the interest to the time of the first payment, and if the payment exceed the interest, deduct the excess from the principal, and cast the interest upon the remainder to the time of the second payment. If the payment be less than the interest, place it by itself, and cast on the interest to the time of the next payment, and so on until the payments exceed the interest; then deduct the excess from the principal, and proceed as before."

A. held a note against B. of $246,24 dated October 12, 1822. March 12, 1823, paid $ 68; what was due, September 24, 1823?

The sum may be prepared by first finding the time from the date of the note to the first payment, and from the first payment to the final payment, and placing them as follows.

| | | Mon. | Days. |
|---|---|---|---|
| March 12, 1823, - - - - | $68,00 | 5 | 0 |
| What was due Sept. 24, 1823? | | 6 | 2 |

Ans. $190,296.

In this example you first cast the interest on the whole sum to the time of the first payment. Now if the first payment had been just the amount of interest then due, the principal would have remained the same, and the interest continued as though nothing had been paid. But in this case there was much more paid than the amount of the interest; and hence the reason of deducting the excess from the principal. After having deducted the *excess from* the principal, you have the amount then *due, and* must cast the interest on it to the time of final *payment.*

A. holds a note against B. of $866, dated June 20, 1820, on which were the following payments. April 20, 1821, $40; Aug. 20, 1821, $462; what was due Jan. 14, 1822?

| | | Mon. | Days. |
|---|---|---|---|
| April 20, 1821, - - - - | $40 | 10 | 00 |
| August 20, 1821, - - - | 462 | 4 | 00 |
| To Jan. 14, 1822. | | 4 | 24 |

Ans. $434,81.

In this example you first cast the interest to the time of the first payment, and find it exceeds the payment, or the payment is not enough to pay the interest; therefore cast on to the time of the next payment, and add this interest to the interest from the date to the first payment; add the two payments together and subtract the interest from their sum; then deduct the excess from the principal, and proceed as in the former example.

A. holds a bond against B. for $1166,666, dated May 1, 1804, upon which the following payments have been made.

| | | Mon. | Days. |
|---|---|---|---|
| 1. December 25, 1804, - - | $166,666 | 7 | 24 |
| 2. July 10, 1805, - - | 16,666 | 6 | 15 |
| 3. September 1, 1806, - | 50,000 | 13 | 21 |
| 4. June 14, 1807, - - | 333,333 | 9 | 13 |
| 5. April 15, 1808, - - | 620,000 | 10 | 1 |
| What will be due upon it Aug. 3, 1809? | | 15 | 18 |

It is convenient in casting interest on bonds, notes, &c. of the above description, to arrange the work as in the following table.

| | *Principal.* | *Time.* | | *Interest.* | *Payments* | *Excess.* |
|---|---|---|---|---|---|---|
| | | Mon. | Days. | | | |
| 1 | $1166,666 | 7 | 24 | $45,499 | $166,666 | $121,167 |
| | 121,167 | | | | | |
| 2 | 1045,499 | 6 | 15 | 33,978 | 16,666 | |
| 3 | 1045,499 | 13 | 21 | 71,616 | 50,000 | |
| 4 | 1045,499 | 9 | 13 | 49,312 | 333,333 | |
| | | | | 154,906 | 399,999 | 245,093 |
| | 245,093 | | | | | |
| 5 | 800,406 | 10 | 1 | 40,153 | 620,000 | 579,847 |
| | 579,847 | | | | | |
| | 220,559 | 15 | 18 | 17,203 | | |

| | |
|---|---|
| The last remainder, | 220,559 |
| Interest from the last payment, | 17,203 |
| Sum due August 3, 1809, | 237,762 |

# COMPOUND INTEREST

Is calculated by adding the interest to the principal at the end of each year, and making the amount the principal for the succeeding year; then the given principal subtracted from the last amount, the remainder will be the compound interest.

What is the compound interest of $ 216,42 for 2 years and 6 months?

216,42
6
12,9852 interest for 1 year.
216,42
229,40 principal for 2d year.
6
13,7640 interest for 2d year.
229,40
243,16 principal for last 6 months.
3
7,2948 interest for last 6 months.
13,76
12,98
34,03 compound interest for 2 years and 6 mo.

The compound interest at 6 per cent of any sum in Federal Money may be found by multiplying the sum by the following numbers.

For 2 years multiply the given sum

| | | | |
|---|---|---|---|
| by | 112,36 | For 7 years, by | 150,3630 |
| 3 years, | 119,1016 | 8 years, | 159,3848 |
| 4 years, | 126,2476 | 9 years, | 168,9478 |
| 5 years, | 133,8225 | 10 years, | 179,0847 |
| 6 years, | 141,8519 | 11 years, | 189,8298 |

Point the product as in simple interest; when it will show the amount of principal and interest for the given umber of years. Subtract the principal from the a-ount, and the remainder will be the compound interest.

When there are months and days, first find the compound interest for the given years; then, for the months and days cast the simple interest on the *amount* for the years, which, added to the amount, will give the answer.

Any sum at compound interest will double in 11 years, 10 months, and 22 days.

What is the compound interest on $246, for 2 years?
Ans. $ 30,40.

What is the compound interest on $236, for 4 years, 7 months, and 6 days? Ans. 72,669.

---

## *Questions.*

What is interest?

How does it appear that it is right to allow interest for money borrowed?

What is the money lent called?

What is the interest or allowance called?

What are the interest and principal added called?

How many kinds of interest are there, and what are they called?

What is simple interest?

What is compound interest?

What is the general rule for interest?

Why does multiplying by the rate give the right answer?

What is the method of casting interest at 6 per cent?

Why do you take one half the even number of months?

Why do you take one sixth of the days?

Why do you place a cipher at the right of half the even number of months, when the days do not amount to 6?

Why do you point off two more places for decimals, in the product than there are decimals in both the factors?

How do you find the interest for any other rate than 6 per cent?

How do you cast the interest on bonds, notes, &c. where partial payments have been made?

Why do you deduct the excess from the principal?

When the payment is less than the interest, why do you cast on to the time of the next payment?

*How do* you cast compound interest?

## *Examples for Practice.*

1. What is the interest of $264,16 for 2 years and 6 months? Ans. $39,624.

2. What is the interest of $64,18 for 3 years, 5 months, and 7 days? Ans. $13,231.

3. What is the interest of $5,811 for 1 year and 11 months? Ans. $0,668.

4. What is the interest of $86 for 9 months? Ans. $3,87.

5. What is the interest of $78,36 for 5 years, 10 months, and 3 days? Ans. $27,465.

6. What is the interest of $246,25 for 1 year, 5 months, and 5 days, at 3 per cent? Ans. $10,568.

7. What is the interest of $812,30 for 2 years, 8 months, and 4 days? Ans. $130,509.

8. What is the interest of $5,19 for 7 months? Ans. $0,181.

9. What is the interest of $1,07 for 3 years, 6 months, and 15 days? Ans. $0,227.

10. What is the interest of $273,51 at 7 per cent, for 1 year, and 10 days? Ans. $19,677.

11. What is the interest of $200,06 for 1 year and 4 days? Ans. $13,337.

12. What is the interest of $0,001 for 16 years and 8 months? Ans. $0,001.

13. What is the interest of $0,0005 for 33 years, and 4 months? Ans. $0,001.

14. What is the interest of $2586974 for 30 years? Ans. $4659553 and 20 cts.

15. Supposing a note of $867,33 dated Jan. 6. 1794, upon which the following payments should be made, viz.

| | |
|---|---|
| 1. April 16, 1797, | $136,44 |
| 2. April 16, 1799, | 319, |
| 3. Jan. 1, 1800, | 518,68 |

*What would* be due July 11, 1801? Ans. $215,103.

16. Supposing there is a note of $100, dated March 20, 1820, upon which were the following payments, viz.

| | |
|---|---|
| Sept. 2, 1820, | $ 7, |
| Feb. 2, 1821, | 30, |
| July 3, 1821, | 20, |
| Dec. 7, 1821, | 18,50 |
| June 30, 1822, | 10, |
| Sept. 1, 1822, | 10, |

What would be due November 16, 1822 ?

Ans. $13,021.

17. Supposing there is a note of $317,19, dated July 12, 1812, on which were the following payments, viz.

| | |
|---|---|
| Oct. 17, 1812, | $61,10 |
| March 20, 1813, | 73,61 |
| Jan. 1, 1815, | 84, |

What was due, Sept. 18, 1816 ? Ans. $144,363.

18. What is the compound interest of $ 246 for 6 years ? Ans. $102,955.

## CONVERSATION VII.

### PROPORTION, OR SINGLE AND DOUBLE RULE OF THREE.

1. *Tut.* I have taught you in the preceding Conversations, the different methods necessary for performing on all numbers, whether whole or fractional, simple or compound, the four fundamental operations of arithmetic, viz. Addition, Subtraction, Multiplication, and Division. But in order to facilitate the application of these rules, it will be necessary to instruct you in other rules, founded on the preceding ; and that which I propose now to consider, is the *single rule of three.* This is sometimes called the golden rule, for on a proper application of it a great part of the business of arithmetic depends.

If 4 yards of cloth cost $8, what will 6 yards of the same cloth cost ?

2. It is plain in this example that if we knew the price of 1 yard, we could find the price of 6 yards by repeating this price 6 times. But as the price of 4 yards is

given, if we multiply 6 by it we shall have 4 times the price of 6 yards, consequently, if we divide this product by 4, the quotient will be the price of 6 yards, viz. $12. Here you see that the price of 4 yards is double the quantity of cloth, and the price of 6 yards must be double its quantity, or $12. If the price of 4 yards had been 3 times the quantity, the price of 6 yards must have been three times that quantity ; and whatever the price of 4 yards may be, the price of 6 yards must be such, that it will contain 6 as often as the price of 4 yards contains 4. The proportion here is, as 4 is to 6 so is 8 to the answer, which is 12. To denote that there is a proportion between the numbers, they are written thus.

4 : 6 :: 8 : 12

that is, 4 is the same part of 6, that 8 is of 12, or 4 is contained in 6 as often as 8 is contained in 12.

3. Again, if it were required to find how many days it would take 12 men to do a piece of work which 8 men, working at the same rate, could do in 16 days, you will see that the days should be less in proportion as the number of men is greater. Hence the proportion is not as 8 is to 12, so is 16 to the answer, but, as 12 is to 8 so is 16 to the answer, thus,

12 : 8 :: 16 : 24.

This last example is called *inverse proportion*, in distinction from the former, which is called *direct proportion*. From what has been said, you will readily understand the following rule for the

## SINGLE RULE OF THREE.

**Write that number, which is of the same kind with the answer, for the third term ; then consider whether the answer ought to be larger or smaller than this number; if it ought to be larger, write the largest of the other two numbers for the second term, and the least for the first; but if it ought to be smaller, write the least of the other two numbers for the second term, and the largest for the first; then multiply the two last terms together, and divide the product by the first term, and the quotient will be the answer.**

If 16 bushels of corn cost \$12, what will 32 bushels cost?

```
16 :: 32 : 12
       12
      ---
       64
      32
16)384(24          Ans. $24.
   32
   ---
    64
    64
```

Here we have the price of 16 bushels given, and are required to find the price of 32 bushels at the same price per bushel. If we knew the price of one bushel, we could multiply 32 by it, and it would give the price of 32 bushels. But here we have the price of 16 bushels, and multiply 32 by it, which gives 16 times the price of 32 bushels, consequently, the quotient arising from the division of this product by 16, is the price of 16 bushels.

If 12 men perform a piece of work in 16 days, in what time will 18 men, working at the same rate, perform the same work?

```
18 :: 12 : 16
       16
      ---
       72
      12
18)192(10 12/18=2/3      Ans. 10 2/3 days.
   18
   ---
    12
```

This is an example of inverse proportion; the number of days decreasing as the number of men increases; consequently the answer ought to be less than the third term, and the smallest of the other two terms, must be the second term.

If, in this example, the time in which one man could perform the work, was given, we could, by dividing that time by the number of men, find how long it would take those men to perform the work. But here we have the time, in which it took 12 men to perform the work, given to find how long it would take 18 men to do the same work. Now it is evident, that it would take one man 12 times as long as it would take 12 men to do the work; therefore, if we multiply 16, the number of days in which 12 men performed the work, by 12, we obtain the time in which it would take one man to perform the work, and this time, divided by 18, gives the time in which it would take 18 men to perform the work, viz. 10⅔ days.

*Pup.* This appears very plain, and from what you have taught me concerning proportion, I should suppose, that almost any question might be solved by it.

*Tut.* This rule is used in almost all cases for transacting business, and you must not leave it till it is perfectly familiar. All the difficulty in these questions consists in stating the proportion. In every question in this rule, there are three numbers given for moving the operation. Two of these numbers are of the same kind, and must be the first and second terms of the proportion. The third number is different from the two first, and of the same kind with the answer. Should the first and second terms consist of different denominations, they must be reduced to the lowest denomination mentioned in either. If the third term consists of more than one denomination, it must be reduced to the lowest denomination mentioned in it, and the answer will ever be in the same denomination in which the third term is.

If 5 cwt. of beef cost $6, what will 9 cwt. 2 qr. 10 lb. cost?

```
        5  :  9  2  10  ::  6
        4     4
       --    --
       20    38
       28    28
      ---   ---
      160   314
       40    76
      ---  ----
      560  1074
              6
    -----------
    560)6444(11,507              Ans. $ 11,507.
        560
        ---
        844
        560
        -----
        28400
        2800
        -----
         4000
         3920
         ----
           80
```

If $12 will buy 10 cwt. 1 qr. of beef, how much will $16 buy?

12 : 16 :: 10 1

4

41

16

246

41

12)656(54     4)54

60

13 2.     Ans. 13cwt. 2qrs. 18$\frac{2}{3}$lb.

56

48

8

28

64

16

224(18$\frac{8}{12}$ = $\frac{2}{3}$

12

104

96

8

4. "Proportion is also applied to questions, in which the relation of the quantity required, to the given quantity of the same kind, depends upon several circumstances combined together; it is then called *Compound Proportion, or Double Rule of Three.*"

If a man travel 240 miles in 10 days, when the days are 12 hours long, how far will he travel in 12 days when they are 16 hours long?

In this example, there are two proportions to be considered, viz. of the days, and the hours in each day, or the number of miles the man travels depends on the number of days he travels, and the number of hours he travels, ıch day. Now if the number of hours had been the ıme in both cases, the proportion would have been,

10 : 12 :: 240 to the answer;

r, if a man in 10 days, travel 240 miles, how far will he avel in 12 days? The answer is 288 miles.

But as the number of hours is different in the two cases, e must substitute another proportion, and say, if a ıan, travelling 12 hours a day for a certain number of ays, will travel 288 miles, how far will he go if he trav-l 16 hours per day? The proportion in this case will be

12 : 16 :: 288 to the answer, 384.

The question may likewise be resolved in this way. y travelling 10 days, when the days are 12 hours long, e travelled 120 hours, and by travelling 12 days, when ıe days are 16 hours long, he would travel 192 hours, hich may be reduced to this proportion,

120 : 192 :: 240 : 384

You will see, by the first mode of resolving the ques-on, that 240 miles has to the fourth term or answer, the ıme proportion that 10 days has to 12 days, and that 12 ours has to 16 hours; stating this in form of proportion, will be,

$$\left.\begin{matrix}10 : 12\\ 12 : 16\end{matrix}\right\} :: 240, \text{ which will give the Ans. 384.}$$

Thus it appears that 240 is to be multiplied by the roduct of 12 by 16, and divided by the product of 10 by 2, or the product of 12 by 16 is to be multiplied by 240, ıd divided by the product of 10 by 12.

Again, if 12 men, working 8 hours a day, spend 24 days ı digging a ditch, 48 feet long, 12 feet wide, and 6 feet eep, how many days will it take 18 men, of the same bility, working 9 hours a day, to dig a ditch 112 feet ıng, 8 feet wide, and 4 feet deep?

This question appears very complicated, but can be re-lved by proportion, as follows,

$$\left.\begin{matrix}18 : 12\\ 9 : 8\\ 48 : 112\\ 12 : 8\\ 6 : 4\end{matrix}\right\} :: 24$$

In this example the answer is to be in days, consequently 24 must be the third term; as 18 men can perform the work in less time than 12 can, 18 must be the first term and 12 the second; and as it will not require so many days to perform the same work when the days are 9 hours long, as it will when they are but 8 hours long, 8 must be the second term, and 9 the first; and as it will take longer to dig a ditch 112 feet long, 8 feet wide, and 4 feet deep, than it will to dig one, 48 feet long, 12 feet wide, and 6 feet deep, 112 must be the second term, and 48 the first.

This question may be stated by first finding the contents of each ditch in feet, when the statement will be,

$$\left.\begin{array}{r} 18 : 12 \\ 9 : 8 \\ 3456 : 3584 \end{array}\right\} :: 24.$$

From what you have been taught, you will readily understand the following rule for the

## DOUBLE RULE OF THREE.

"Make the number, which is of the same kind with the required answer, the third term; and of the remaining numbers, take any two that are of the same kind, and place one for a first term, and the other for a second term, according to the directions in simple proportion; then, any other two of the same kind, and so on, till all are used; lastly, multiply the third term by the product of the second terms, and divide the result by the product of the first terms, and the quotient will be the fourth term, or answer required."

*Pup.* I think that I understand this sufficiently to perform any question belonging to the double rule of three.

*Tut.* You will frequently find questions in this rule which will require close thought to state them correctly. The only difficulty will be in *stating the question*, and this should excite you to obtain a thorough knowledge of the rule.

If a man travel 130 miles in 3 days, when the days are hours long, in how many days, of 10 hours each, may travel 360 miles ?

130 : 360 :: 3
10 : 12

1300 4320
3

$1300)12960(9\frac{63}{65}$ days, *or* 9,97 days.
11700

$\frac{1260}{1300}=\frac{63}{65}$

---

### *Questions.*

What is the single rule of three ?
What is the rule for it ?
Why do you multiply the second and third terms tether, and divide by the first ?
When the answer is to be less than the third term, hich of the other two terms must be the second term ?
When the first and second terms consist of different enominations, what do you do ?
When the third term consists of different denominaons, what denomination will the answer be in ?
What is compound proportion, *or* double rule of three ?
What is the rule ?
How does it appear that this will give the true answer ?

◆

### *Examples for Practice.*

1. If 100 workmen can finish a piece of work in 12 ays, how many are sufficient to do the same work in days ? Ans. 400.

2. If 6 horses eat 21 bushels of oats in 3 weeks, how *any* bushels will 20 horses eat in the same time ? Ans. 70 bushels.

3. If 20 horses eat 70 bushels of oats in 3 weeks, how many bushels will 6 horses eat in the same time?
Ans. 21 bushels

4. If the moon move 13 deg. 10 min. 35 sec. in one day, in what time does it perform one revolution?
Ans. 27d. 7h. 43m.

5. If 365 men consume 75 barrels of provisions in 9 months, how much will 500 men consume in the same time? Ans. 102$\frac{54}{73}$ barrels.

6. If I give $6 for the use of $100 for 12 months, what must I give for $357,82 the same time? Ans. $21,469.

7. How many tiles of 8 inches square will lay a floor 20 feet long, and 16 feet broad? Ans. 720.

8. If I buy 76 yards of cloth for $113,17, what did it cost per Ell English? Ans. $1,861.

9. If 3 horses and 4 oxen be worth 9 cows, how many cows will 6 horses and 8 oxen be worth? Ans. 18.

10. If a staff, 5 ft. 8 in. in length, cast a shadow of 6ft. how high is that steeple whose shadow measures 153 ft.?
Ans. 144$\frac{1}{2}$ ft.

11. Bought 4 pieces of Holland, each containing 24 Ells English, for $96; how much was that per yard?
Ans. $0,80.

12. A bankrupt owes in all $920, and his money and effects are but $607,50; what will a creditor receive on $11,333? Ans. $ 7,083.

13. Bought 126 gallons of rum for $110, how much water must be added to it, to reduce the first cost to 75 cents per gallon? Ans. 20$\frac{2}{3}$ gal.

14. If a person, whose rent is $145, pays $12,63 of taxes, how much should a person pay whose rent is $378?
Ans. $32,925.

15. If a board be 9 inches broad, how much in length will make a square foot? Ans. 16 inches.

16. How many yards of sarsenet, 3 qrs. wide, will line 9 yards of cloth of 8qrs. wide? Ans. 24 yards.

17. Lent a friend 292 dollars for 6 months; some time afterwards he lent me 806 dollars; how long may I keep it to balance the favour? Ans. 2 months, 5 days.

18. How many yards of matting, 2 ft. 6 in. broad, will cover a floor, that is 27 ft. long, and 20 ft. broad? Ans. 72.

19. How many yards of cloth, 3 qrs. wide, are equal in measure to 30 yards, 5 qrs. wide? Ans. 50 yards.

20. How many yards of canvass that is an ell wide, will line 20 yards of say that is 3 qrs. wide? Ans. 12.

21. A wall, that is to be built to the height of 27 ft. was raised 9 feet by 12 men in 6 days; how many men must be employed to finish the wall in 4 days, at the same rate of working? Ans. 36.

22. If, when the days are 13$\frac{5}{8}$ hours long, a traveller perform his journey in 35$\frac{1}{2}$ days; in how many days will he perform the same journey, when the days are 11$\frac{9}{10}$ hours long? Ans. 40$\frac{615}{952}$ days.

23. How many yards of cloth may be bought for $195,75 of which 9$\frac{1}{4}$ yards cost $11,02? Ans. 168yds. 3qrs.

24. If I buy 57 yards of cloth for 49 guineas, at 28 shillings each; what did it cost per Ell English? Ans. $5,012.

25. If the distance from Worcester to Templeton be 31 miles; I demand how many times a wheel, whose circumference is 15$\frac{1}{2}$ feet, will turn round in performing the journey? Ans. 10560 times.

26. What will 37$\frac{1}{2}$ gross of buttons come to at 13 cents per dozen? Ans. $58,50.

27. A farm, containing 125 acres, 3r. 27pls. is rented at $11,50 per acre; what is the yearly rent of that farm? Ans. $1447,065.

28. If $\frac{7}{16}$ of a ship cost $1163, what is the whole worth? Ans. $2658,285.

29. A merchant bought 9 packages of cloth at 3 guineas for 7 yards; each package contained 8 parcels, each parcel 12 pieces, and each piece 20 yards; how many *dollars* came the whole to, and how much per yard? Ans. $34560 the whole; $2 per yd.

30. A merchant bought 49 tuns of wine for $910; freight cost $90; duties, $40; cellar, $31,67; other charges $50; and he would gain $185 by the bargain; what must I give him for 23 tuns? Ans. $613,33.

31. The earth being 360 degrees in circumference, turns round on its axis in 24 hours; how far does it turn in one minute, in the 43d parallel of latitude; the degree of longitude, in this latitude, being about 51 statute miles?
Ans. 12$\frac{3}{4}$ minutes.

32. There is a cistern having 4 cocks; the first will empty it in ten minutes; the second in 20 minutes; the third in 40 minutes; the fourth in 80 minutes; in what time will all four running together empty it?

$$\text{As}\left\{\begin{matrix}10\\20\\40\\60\end{matrix}\right\}\begin{matrix}\overset{\text{Min.}}{:\ 60}\\:\ 80\end{matrix}::\overset{\text{Cist.}}{1}\begin{array}{c}\left\{\begin{matrix}6\\3\\1\frac{1}{2}\\\frac{3}{4}\end{matrix}\right\}\\\hline 11\frac{1}{4}\end{array}\quad\text{As }\overset{\text{Cist.}}{11\tfrac{1}{4}}:\overset{\text{Min.}}{60}::\overset{\text{Cist.}}{1}:\overset{\text{Min.}}{5\tfrac{1}{3}}\text{ Ans.}$$

33. A. and B. depart from the same place, and travel the same road; but A. goes 5 days before B. at the rate of 20 miles per day; B. follows at the rate of 25 miles per day; in what time and distance will he overtake A?
Ans. 20 d. and 500 mls.

34. A merchant bought a number of bales of velvet, each containing 129$\frac{17}{27}$ yards, at the rate of $7 for 5 yards, and sold them out at the rate of $11 for 7 yards; and gained $200 dollars by the bargain; how many bales were there? Ans. 9 bales.

35. Suppose 450 men are in a garrison, and their provisions are calculated to last but 5 months; how many must leave the garrison, that the same provisions may last those who remain, 9 months? Ans. 200 men.

36. If a piece of land, 40 rods in length, and 4 in breadth, make an acre, how wide must it be, when it is but 19 rods long, to make an acre? Ans. 8 r. 6 ft. 11$\frac{7}{19}$ in.

37. If 9 men can build a house in 5 months, by working 14 hours per day, in what time will the same numb[illegible] of men do it, when they work only 10 hours per day?
Ans. 7 months.

38. What number of men must be employed to finish *in 9 days*, what 15 men would be 30 days about? Ans. 50.

39. If a field will feed 6 cows 91 days, how long will it feed 21 cows? Ans. 26 days.

40. A regiment of soldiers, consisting of 745 men, is to be clothed, each suit is to contain $3\frac{1}{2}$ yards of cloth, which is $1\frac{3}{8}$ yard wide, and lined with shalloon $\frac{7}{8}$ of a yard wide; how many yards of shalloon will line them?
Ans. $4097\frac{1}{2}$ yards.

41. What quantity of water must I add to a pipe of wine, valued at $110, to reduce the first cost to $0,75?
Ans. $20\frac{2}{3}$ gallons.

42. If a person, having $\frac{3}{5}$ of a coal mine, sells $\frac{3}{4}$ of his share for $570, what is the whole mine worth?
Ans. $1266,666.

43. A. borrowed of his friend $833,333 for 7 months, promising to do him the like kindness; sometime after, B. had occasion for $1000, how long may he keep it to receive full amends for the favour?
Ans. 5 months and 25 days.

44. If the keeping of one horse be 16 cents per day, what will be that of 11 horses for a year? Ans. $642,40.

45. Bought 3 casks of raisins, each weighing 2 cwt. 2 qrs. 25 lb. what will they come to at $6,943 per cwt?
Ans. $56,721.

46. If an ingot of gold, weighing 9 lb. 9 oz. 12 pwt. be worth $1372, what is that per grain? Ans. $0,024.

47. If I borrowed 185 quarters of corn when the price was $3,166, how much must I pay to indemnify the lender, when the price is $2,89? Ans. $202\frac{1930}{2890}$.

48. If my horse and saddle are worth 18 guineas, and my horse be worth six times as much as my saddle, what is the value of my horse? Ans. $72.

49. If 120 bushels of corn will serve 14 horses 56 days; how many days will 94 bushels serve 6 horses?
Ans $102\frac{16}{45}$ days.

50. If the transportation of 13cwt. 1qr. 72 miles be [illegible],416, what will be the transportation of 7cwt. 3[illegible] 112 miles? Ans. $7,395.

51. If a regiment of soldiers, consisting of 939 men, *onsume 351* quarters of wheat in 7 months; how many

soldiers will consume 1464 quarters in 5 months, at that rate? Ans. $5483\frac{23}{193}$.

52. If 248 men in 5 days, of 11 hours each, dig a trench 230 yards long, 3 wide and 2 deep, in how many days of 9 hours long, will 24 men dig a trench of 420 yards long, 5 wide and 3 deep? Ans. $288\frac{59}{207}$.

53. If the freight of 12 cwt. 2qr. 6lb. 275 miles cost $27,78; how far may 60cwt. 3qr. be shipped for $234,78? Ans. 480 miles.

54. If the freight of 9hhd. of sugar, each weighing 12cwt. 20 leagues, cost $53,33; what must be paid for 50 tierces ditto, each weighing 2½cwt. 100 leagues? Ans. $308,62.

55. There was a certain edifice completed in a year by 20 workmen; but the same being demolished, it is necessary that just such an one should be built in 5 months. I demand the number of men to be employed about it? Ans. 48 men.

## CONVERSATION VIII.

### FELLOWSHIP AND LOSS AND GAIN.

*Tut.* What is fellowship?

*Pup.* I cannot tell, unless it is a kind of partnership.

*Tut.* Fellowship is a rule by which traders, and others doing business in company, determine each partner's particular share of the gain or loss, according to the proportion of his share to the joint stock or capital.*

Fellowship is of two kinds, *single* and *double*.

Single fellowship is when the stocks of the partners are employed equal times.

A. B. and C. put $1000 each, into trade; at the end of one year, they find they have gained $1200; what is each partner's share of the gain?

---

* In commercial language, the sum of all the money or property which the partners put into trade is called the *stock* or *capital*; the *gain to be divided* is called the *dividend*.

Now the sum of the stocks is $3000, and the whole gain is $1200; and each partner's gain ought to be such that it will have the same proportion to the whole gain, that his share of the whole stock, has to the whole stock; thus the proportion will be,

3000 : 1000 :: 1200.

Hence the reason of the following rule for,

## SINGLE FELLOWSHIP.

**As the whole stock is to each partner's share of the stock, so is the whole gain or loss to his share of the gain or loss.**

A. B. and C. companied; A. put in $400, B. $800, and C. 1200, and they gained $600; what was each partner's share of the gain?

1200 2400 : 600 :: 400 : 100 A's share.
800 2400 : 600 :: 800 : 200 B's share.
400 2400 : 600 :: 1200 : 300 C's share.

Whole stock 2400

*Pup.* This is very plain, for I see nothing in it different from the single rule of three.

*Tut.* There is nothing different, except in preparing the question for statement, which you will readily understand from the example given above.

---

Double fellowship is when the stocks are employed unequal times.

A. and B. enter into partnership; A. puts in $400 for 4 months; B. puts in $800 for 8 months; at the end of the trade they have gained $200; what is each partner's share?

Now had the stocks been equal, B. would have had double the share of A. because he continued his stock in trade double the time of A.; and had the times been equal, B. would have had double the share of A. because he had double the stock of A. But as B. had double the *stock* of A. and continued it in trade double the time of A. *he* must have 4 times as much of the gain as A. The stock of A. multiplied by the time it was continued in trade gives a product of 1600; B's stock multiplied by the time it was continued in trade, gives a product of 6400. 6400 is 4 times 1600, and as B's share of the gain ought to be 4 times as much as A's share, the proportion will be, as the sum of these products is to each partner's product, so is the whole gain to each partner's share of the gain, thus,

6400    8000 : 1600 :: 200 : 40 A's share.
1600    8000 : 6400 :: 200 : 160 B's share.
————
8000

Hence the following rule for

## DOUBLE FELLOWSHIP.

**Multiply each partner's share of the stock by the time it was continued in trade. Add together these several products; then say, as the sum of these products is to each partner's product, so is the whole gain to each partner's share of the gain.**

A. and B. hold a piece of ground in common, for which they are to pay $36. A. put in 23 oxen for 27 days, and B. 21 oxen for 35 days; what part of the rent ought each man to pay?

23 × 27 = 621    1356 : 621 :: 36
21 × 35 = 735    36
————
1356

3726
1863
————
1356)22356(16,48 &c. A's share.
1356
————
8796
8136
————
6600
5424
————
11760
10848
————
912

1356 : 735 :: 36 : 19,51 &c. B's share.

Three graziers hired a piece of land for $60,50. A. put in 5 sheep for $4\frac{1}{2}$ months, B. put in 8 for 5 months, and C put in 9 for $6\frac{1}{2}$ months; how much must each pay of the rent? Ans. A. $11,25, B. $20, and C. $29,25.

# LOSS AND GAIN.

Loss and Gain is a rule by which merchants estimate the profit or loss in buying and selling goods, and raise or lower the price of them so as to gain or lose so much per cent.

To know what is gained or lost per cent; *First, find what the gain or loss is by subtraction; then, as the price it cost is to $100, so is the gain or loss to the gain or loss per cent.*

If I buy corn for $0,75 per bushel, and sell it again for $0,80 per bushel, what do I gain per cent, or in laying out $100?

Sold at $0,80 per bushel,
Bought at 0.75 per bushel.

$0,05 gain per bushel.

Then, as ,75 : 100,00 :: 05
05

,75)50000(6,66$\frac{50}{75}$ = $\frac{10}{15}$. Ans. $6,66$\frac{10}{15}$.
450

500
450

500
450

50

If I buy cloth at $1,02 per yard, and sell it again at $0,90 per yard; what do I lose per cent, or in laying out $100? Ans. $11,765.

If I buy 37 gallons of brandy at $1,10 per gallon, and sell it for $40, what do I gain or lose per cent?
Ans. $1,719 loss.

---

To know how a commodity must be sold, to gain or lose so much per cent;—*As* $100 is to $100 *with the profit added, or loss subtracted, so is the price which the commodity cost to the gaining or losing price.*

If I buy corn for 80 cents per bushel, how must I sell it so as to lose 15 per cent?

$ cts.
100 : ,85 :: 80
80 Ans. $0,68 per bushel.
100)68,00
,68

If I buy cloth at $2,50 per yard; how must I sell it so as to lose 17½ per cent? Ans. $2,062½.

◆

*Questions.*

What is fellowship?
Of how many kinds is fellowship?
What is single fellowship?
What is the rule?
How does it appear that this will give a right answer?
What is double fellowship?
What is the rule for it?
Why does this give the right answer?
What is loss and gain?
How is the loss or gain found when the price of the commodity is given?
Why should this give the answer?
How do you proceed when you wish to know how a commodity must be valued to gain or lose so much per cent?
How does it appear that this will give the true answer?

### *Examples for Practice.*

1. A. and B. have gained by trading, $182. A. put into stock $300, and B. $400; what is each person's share of the profit? Ans. A. $78, and B. $104.

2. Three merchants, A. B. and C. freight a ship with 340 tuns of wine; A. loaded 110 tuns, B. 97, and C. the rest. In a storm the seamen were obliged to throw 85 tuns overboard; how much must each sustain of the loss? Ans. A. $27\frac{1}{2}$, B. $24\frac{1}{4}$, and C. $33\frac{1}{4}$.

3. A bankrupt is indebted to A. $277,33; to B. $305,17; to C. $152, and to D. $105. His estate is worth only $677,50; how must it be divided?

Ans. A. $\$223,81\frac{2580}{8395}$, B. $\$246,28\frac{615}{8395}$, C. $\$122,66\frac{6930}{8395}$, D. $\$84,73\frac{6665}{8395}$.

4. A. and B. venturing equal sums of money, clear by joint trade, $154. By agreement, A. was to have 8 per cent, because he spent his time in the execution of the project, and B. was to have only 5 per cent; what was A. allowed for his trouble? Ans. $\$35,53\frac{11}{13}$.

5. A man died leaving 3 sons, to whom he bequeathed his estate in the following manner, viz. to the eldest he gave $184, to the second $155, and to the third $96; but when his debts were paid, there were but $184 left; what is each one's proportion of his estate?

Ans. $77,829 }
65,563 } Shares.
40,606 }

6. A. and B. companied; A. put in $120, and took $\frac{5}{9}$ of the gain; what did B. put in? Ans. $96.

7. Four merchants traded in company; A. put in $400 for 5 months; B. $600 for 7 months; C. $960 for 8 months; and D. $1200 for 9 months; but by misfortunes at sea, they lost $750; what must each man sustain of the loss?

Ans. A. $\$94,936\frac{56}{79}$, B. $\$142,405\frac{5}{79}$, C. $\$227,848\frac{8}{79}$, D. $\$284,810\frac{10}{79}$.

8. Two merchants enter into partnership for 18 months; A. put into stock at first $200, and at the end of 8 months, he put in $100 more; B. put in at first $550,

and at the end of 4 months took out $140. Now at the expiration of the time they find they have gained $526; what is each man's just share?

Ans. A.'s, $192,95$\frac{70}{1254}$, B.'s, $333,04$\frac{1184}{1254}$.

9. A. with a capital of $1000, began trade January 1, 1820, and meeting with success in business, he took in B. a partner, with a capital of $1500, on the first of March following. Three months after that, they admit C. as a third partner, who brought into stock $2800, and after trading together till the first of the next year, they find the gain, since A. commenced business, to be $1776,50. How must this be divided among the partners?

Ans. A's, $457,40$\frac{364}{466}$. B's, 571,83$\frac{222}{466}$. C's, 747,19$\frac{346}{466}$.

10. A draper bought 60 yards of cloth at $4,50 per yard, 38 yards of cloth at $2,50 per yard, and sold them one with another, at $4,25 per yard; d[illegible]he gain or lose, and what per cent? Ans. gained $14,11 per cent.

11. If I buy an article at $1,20 per pound, and sell it at 90 cents per pound, what do I lose per cent? Ans. $25.

12 If a barrel of potash cost $15, how must it be sold to lose 10 per cent? Ans. $13,50.

13. Bought 30 hogsheads of molasses at $600; paid in duties $20,66; for freight $40,78, for porterage $6,05, and for insurance $30,84; if I sell it at $26 per hogshead, how much shall I gain per cent? Ans. $11,695.

# CONVERSATION IX.

## DISCOUNT AND EQUATION OF PAYMENTS.

*Tut.* What is discount?

*Pup.* Discount, I believe, is an allowance made for the payment of a debt before it becomes due.

1. *Tut.* Discount is an allowance made by the creditor to the debtor, for the payment of a debt before it becomes due, and is the difference between that sum, due sometime hence, and its present worth. If A. owe B.

$500, payable in 6 months, with interest, after it becomes due, he would, by paying the debt before the 6 months were out, confer a favour on B. for which B. ought to make an allowance, and this allowance is the *discount.*

2. By the present worth, is meant such a sum, which, put at interest, would in the given time, and at the given rate, amount to the debt then due.

3. It is very evident that an allowance ought to be made for paying a debt before it is due; for if the debtor keep the money in his own hands, he can employ it to his own benefit, while it is of no benefit to the creditor; but if he pay the debt before it is due, he relinquishes this privilege to the creditor, hence the reason of making a discount.

4. Now the present worth is such a sum, which if put to interest, would, in the given time, and at the rate per cent. for which the discount is to be made, amount to the sum or debt then due. At first you might suppose the discount to be the interest on the sum or debt, from the time at which it is paid, to the time at which it is due. But this is not true, for if you cast the interest on the whole debt from the time it is paid to the time when it would become due, and subtract this interest from the whole debt, the remainder, put at interest for the same time and rate, will not amount to the debt, therefore the discount is not so much as the interest on the sum, for the time and rate for which the discount is to be made, but the discount is such a sum, which will be equal to the interest on the present worth for the given time and rate.

Hence the following rule for

# DISCOUNT.

**As the amount of $100 for the given rate and time is to the given sum or debt, so is $100 to the present worth.**

**or,**

**As the amount of $100 for the given rate and time is to the given sum or debt, so is the interest of $100, for the same time, to the *discount.***

What is the present worth and discount of $350, payable in half a year, discounting at 6 per cent. per annum?

100
3
$3,00 interest for 1 year.
100
103 : 350 :: 100
100
103)35000(339,805
309
410
309
1010
927
830
824
600
515
85

103 : 350 :: 3
3
103)1050(10,194
103
200
103
970
927
430
412
18

Ans. { Present worth, $339,805.
Discount, $10,194.

*Pup.* I see nothing very difficult in this, and think that I shall be able to perform any question belonging to this rule.

## EQUATION OF PAYMENTS.

*Tut.* What is equation of payments?

*Pup.* *Equation* of *payments* is the finding a time to pay at once, several debts due at different times, so that neither party shall sustain loss.

*Tut.* You are right. The following is the rule by which payments are generally equated.

**Multiply each payment by the time at which it is due; then divide the sum of the products by the sum of the payments, and the quotient will be the time required.**

This rule is founded on the supposition, that the sum of the interests of the several debts, which are payable before the equated time, from their terms to that time, ought to be equal to the sum of the interests of the debts payable after the equated time, from that time to their terms. But this is not strictly true; for by keeping a debt after it is due, you save the interest; but by paying a debt before it is due you only lose the discount, which is not so much as the interest; ( 4, Con. IX.) therefore the rule is not true. But the error is so small that this method will always be used as the most eligible in common business.

A. owes B. $400 to be paid in 4 months, and $400 more to be paid in 8 months; what is the equated time for paying the whole?

It is evident the time must be 6 months, for the two debts are equal, and the equated time must be half the sum of both the times. 400 multiplied by 4, is 1600; and 400 multiplied by 8, is 3200, the sum of which is 4800, which, divided by 800, the sum of the two payments, gives 6 months, the equated time.

A. owes B. $380, to be paid as follows; $100 in 6 months; $120 in 7 months; and $160 in 10 months, what is the equated time for the payment of the whole debt?

$$\begin{array}{lll}
100 \times 6 = 600 & & \\
120 \times 7 = 840 & & \\
160 \times 10 = 1600 & & \\
\hline
380 & ) & 3040(8 \text{ months, Ans.} \\
 & & 3040 \\
\hline
\end{array}$$

### *Questions.*

What is discount?

What is meant by the present worth of any sum?

Why do the creditors ought to allow discount for the payment of a debt before it is due?

What is the rule for discount?

Why should not the discount be the same as the interest on the sum for the given rate and time?

What is the rule for equation of payments?

Is this rule founded on correct principles?

Where is the error?

---

### *Examples for Practice.*

1. What ready money will discharge a debt of $1595, due 5 months and 20 days hence, at 6 per cent?
Ans. $1541,326.

2. What is the present worth of $960, payable as follows, ½ at 3 months, ⅓ at 6 months, and the rest at 9 months, supposing the discount to be made at 6 per cent?
Ans. $936,70.

3. Bought a quantity of goods for $250, and sold them for $300 payable 9 months hence; what was the gain, supposing the discount to be made at 6 per cent?
Ans. $37,081¼.

4. What is the present worth of $426,55 to be paid 8 months hence, at 6 per cent?

5. What is the present worth of $426 payable in 4 years and 12 days, at 5 per cent? Ans. $354,519.

6. The sum of $164,166 is to be paid, ½ in 6 months, ⅓ in 8 months, and ⅙ in 12 months; what is the mean time for the payment of the whole? Ans. 7⅔ mo.

7. A debt is to be paid as follows; ¼ at 2 months, ⅛ at 3 months, ⅛ at 4 months, ⅛ at 5 months, and the rest at 7 months; what is the equated time to pay the whole?
Ans. 4 months and 18 days.

8. A merchant owes me $900, to be paid in 96 days; $130 in 120 days, $500 in 80 days, $1267 in 27 days; what is the mean time for the payment of the whole?
Ans. 63 days, very nearly.

# CONVERSATION X.

## SINGLE AND DOUBLE POSITION.

*Tut.* What is position?

*Pup.* Position is a rule relating to false or supposed numbers.

*Tut.* Position is a method of performing, by false or supposed numbers, such questions as cannot be resolved by the common direct rules. It is of two kinds, *single* and *double.*

## SINGLE POSITION

Shows how to resolve, by the working of one supposed number, as if it were a true one, those questions, whose results are proportional to their suppositions.

The following is the rule for *single position.*

Take any number and perform the same operations with it as are described to be performed in the question. Then say, as the sum of the errors is to the supposed number, so is the given sum to the true one required.

A. B. and C. talking of their ages, B. said his age was once and a half the age of A.; and C. said his age was twice and one tenth the age of both, and that the sum of their ages was 93; what was the age of each?

Suppose A.'s age is 20
B.'s must be 30
and C.'s 105

155

Then as 155 : 20 :: 93
20
155)1860(12 A.'s age.
155
310
310

155 : 30 :: 93
30
155)2790(18 B.'s age.
155
1240
1240

155 : 105 :: 93
93
315
945
155)9765(63 C.'s age.
930
465
465

Ans. A.'s age 12 years.
B.'s age 18 years.
C.'s age 63 years.

*Pup.* It appears very plain that the proportion between the *sum* of the supposed ages and each of the supposed ages respectively, must correspond with the proportion between the sum of the true ages and each of the true ages, respectively.

---

*Tut.* Do you know in what double position differs from single position ?

*Pup.* I do not ; but expect it is more difficult than single position, and should think from the name that there must be more than one supposition for the same question.

*Tut.* You are correct ; for it is more difficult, and to every question there must be two suppositions. *Double Position* shows how to resolve such questions as require two suppositions. The following is the rule for

## DOUBLE POSITION.

Take any two numbers, and proceed with them according to the conditions of the question. Place the errors against their respective positions or supposed numbers, and if the error be too large, mark it with † ; but if too small with —. Multiply the first supposed number by the last error, and the last supposed number by the first error. If the errors be alike, that is, both too large, or both too small, divide the difference of the products by the difference of the errors, and the quotient will be the answer ; but if the errors be unlike, that is, one too large and the other too small, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

This rule is founded on the principle, that the first error is to the second, as the difference between the first supposed number and the true number, is to the difference between the second supposed number and the true number.

What number is that, which, being increased by its $\frac{1}{2}$, its $\frac{1}{4}$, and 5 more, will be doubled?

| | | | |
|---|---|---|---|
| Suppose it is | 16 | Suppose it is | 12 |
| One half is | 8 | One half is | 6 |
| One quarter is | 4 | One quarter is | 3 |
| | 5 | | 5 |
| | 33 | | 26 |
| Double the sup. num. | 32 | Double the sup. num. | 24 |
| | 1— | | 2— |

$$16 \times 2 = 32$$
$$12 \times 1 = 12$$
$$2 - 1 = 1)20$$
$$20 \text{ Ans.}$$

A labourer was hired for 60 days upon this condition, that for every day he wrought, he should receive 75 cents; and for every day he was idle he should forfeit 37½ cents; at the expiration of the time he received $18; how many days did he work, and how many was he idle?

Ans. He wrought 36 days, and was idle 24.

### *Questions.*

What is position?
What is single position?
What is the rule for working single position?
Why does this give the true answer?
What is double position?
What is the rule for double position?
On what principle is this rule founded?

### *Examples for Practice.*

1. A man lent his friend a sum of money unknown, to receive interest for the same at 6 per cent. per annum, simple interest, and at the end of 12 years, received for principal and interest $860; what was the sum lent? Ans. $500.

2. A. B. C. and D. spent $18, at a reckoning, and being a little dipped, they agreed that A. should pay $\frac{2}{3}$, B. $\frac{1}{2}$, C. $\frac{1}{3}$, and D. $\frac{1}{4}$; what did each pay in the above proportion?

Ans. $6,857$\frac{12}{84}$ A's share.
5,142$\frac{72}{84}$ B's share.
3,428$\frac{48}{84}$ C's share.
2,571$\frac{36}{84}$ D's share.

Proof, $18,000

3. A person, having spent $\frac{1}{2}$ and $\frac{1}{3}$ of his money, had $180 left; what had he at first? Ans. $1080.

4. A vessel has 3 cocks, A. B. and C.; A. can empty it in $\frac{1}{2}$ an hour, B. in $\frac{1}{4}$ of an hour, and C. in $\frac{1}{3}$ of an hour; in what time will they all empty it together?

Ans. $\frac{1}{9}$ of an hour.

5. What number is that, which, being increased by $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ of itself, the sum shall be 125? Ans. 60.

6. Seven eighths of a certain number exceeds four fifths by 6; what is that number? Ans. 80.

7. A gentleman has two horses of considerable value, and a carriage worth $400, now if the first horse be harnessed in it, he and the carriage together will be triple the value of the second; but if the second be put in, they will be 7 times the value of the first; what is the value of each horse? Ans. One, $80, the other $160.

8. Suppose the sea serpent's head is 9 inches long, and his tail is as long as his head, and half the length of his body, and his body is as long as his head and tail; what is the whole length of the *monster*? Ans. 6 ft.

9. A farmer having driven his cattle to market received for them all $320, being paid at the rate of $24 per ox, $16 per cow, and $6, per calf; there were as many oxen as cows, and 4 times as many calves as cows; how many were there of each sort?

Ans. 5 oxen, 5 cows, and 20 calves.

10. A. and B. laid out equal sums of money in trade; A. gained a sum equal to $\frac{1}{4}$ of his stock, and B. lost $225; then A.'s money was double that of B.'s; what did each one lay out? Ans. $600.

11. A. B. and C. built a ship which cost them $5000, of which A. paid a certain sum, B. paid $500 more than A., and C. $500 more than both; having finished her, they fixed her for sea, with a cargo worth twice the value of the ship. The outfits and charges of the voyage, amounted to $\frac{1}{8}$ of the ship; upon the return of which, they found their clear gain to be $\frac{2}{3}$ of $\frac{3}{5}$ of the vessel, cargo and expenses; what did the ship cost them, severally; what share had each in her; and what, upon the final adjustment of their accompts, had they severally gained?

Ans. A. owned $\frac{7}{40}$ of the ship, which cost him $875, and his share of the gain was $1093,75. B. owned $\frac{11}{40}$, which cost him $1375, and his gain was $1718,75. C. owned $\frac{11}{20}$, which cost $2750, and his gain was $3437,50.

# CONVERSATION XI.

## INVOLUTION AND EVOLUTION.

*Tut.* What is involution?

*Pup.* I do not know, unless it is multiplying a number by itself.

*Tut.* You have a correct idea of it. Involution is the multiplying any given number into itself continually, a certain number of times. The numbers which are produced in this way are called *powers of the given number.* Thus, if 3 is the given number, 3 will be the first power.

$3 \times 3 = 9$, the second power or square of $3 = 3^2$.
$3 \times 3 \times 3 = 27$, the third power or cube of $3 = 3^3$.
$3 \times 3 \times 3 \times 3 = 81$, the 4th power or biquadrate of $3 = 3^4$.

The small figure placed at the right of the given number, shows the order of the power, and is called the *index*, or the *exponent*.

To find any power, multiply the given number or first power, continually by itself, till the number of multiplications be 1 less than the index or exponent of the power to be found, and the last *product will* be the power required.

What is the fourth power of 6?

$$\begin{array}{rl} 6 & \\ 6 & \\ \hline 36 & \text{2d. power.} \\ 6 & \\ \hline 216 & \text{3d. power.} \\ 6 & \\ \hline 1296 & \text{4th. power or answer} = 6^4. \end{array}$$

| | |
|---|---|
| What is the second power of 45? | Ans. 2025. |
| What is the third power of 3,5? | Ans. 42,875. |
| What is the second power of $\frac{2}{3}$? | Ans. $\frac{4}{9}$. |

# EVOLUTION.

What is evolution?

*Pup.* Evolution is, I think, the reverse of involution, or it is the finding of the number from which the power was raised.

*Tut.* Evolution is the extraction of roots. The root of any given number or power, is the number whose continual multiplication into itself produces the power, and is denominated the square, cube, biquadrate, or 2d, 3d, 4th root, &c accordingly as it is, when raised to the second, third, &c. power, equal to that power. Thus 3 is the square root of 9, because $3 \times 3 = 9$, and 3 is the cube root of 27, because $3 \times 3 \times 3 = 27$, and so on with the other roots.

There is no number, of which we cannot find any *power* exactly; but there are many numbers, of which the exact *roots* can never be obtained. Yet by the aid of decimals we can obtain these roots to any necessary degree of exactness. Those roots which cannot be exactly obtained, are called *surd* or *irrational roots*, in distinction from those which can be exactly obtained, which are called *rational roots.*

The square root is denoted by this character $\sqrt{}$, with the index of the root against it, except when the square root is denoted, the character itself denoting the square root. Thus the square root of 24 is expressed $\sqrt{24}$; the cube root of 36 is $\sqrt{}^{3}\,36$, and so on.

The second, third, fourth, and fifth powers of the nine digits are expressed in the following

## TABLE.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Roots, | or 1st Powers | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Squares, | or 2d Powers, | 1 | 4 | 9 | 16 | 25 | 36 | 49 | 64 | 81 |
| Cubes, | or 3d Powers, | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 | 729 |
| Biquadrates, | or 4th Powers, | 1 | 16 | 81 | 256 | 625 | 1296 | 2401 | 4096 | 6561 |
| Sursolids, | or 5th Powers, | 1 | 32 | 243 | 1024 | 3125 | 7776 | 16807 | 32768 | 59049 |

## TO EXTRACT THE SQUARE ROOT.

The extraction of the square root of any number, is the finding another number, which, multiplied into itself, will produce the given number.

### *Rule.*

"Distinguish the given number into periods of two figures each, by putting a point over the place of units, another over the place of hundreds, and so on, which points show the number of figures the root will consist of.

"Find the greatest square number in the first or left hand period, place the root of it at the right hand of the given number, (after the manner of a quotient in division) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the next period for a dividend.

"Place the double of the root already found, on the left hand of the dividend for a divisor. Seek how often the divisor is contained in the dividend, (except the right hand figure) and place the answer in the root for the second figure of it, and likewise on the right hand of the divisor; multiply the divisor with the figure last annexed, by the figure last placed in the root, and subtract the product from the dividend; to the remainder, join the next *period for* a new dividend.

"Double the figures already found in the root for a new divisor, (or bring down your last divisor for a new one, doubling the right hand figure of it,) and from these, find the next figure in the root, as last directed, and continue the operation in the same manner till you have brought down all the periods."

It frequently happens that the left hand period is not a full period, but, notwithstanding, it must be considered the first period. When there are decimals in the given number, it must be pointed both ways from the place of units. If, when there are integers, the first decimal period be deficient, it may be completed by annexing so many ciphers as the power requires.

To extract the root of a vulgar fraction; *reduce it to a decimal, and proceed as directed above.*

What is the square root of 676?

```
   676(26 Ans.
   4
   ---
46)276
   276
   ---
```

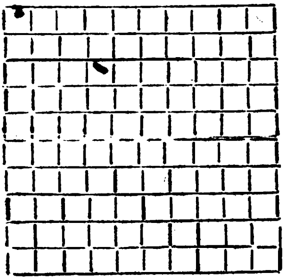
*Pup.* I understand how the work is done, but do not understand the reasons of it; I should like an explanation of the whole work.

*Tut.* "The superficial contents of any thing, that is, the number of square feet, yards or inches, &c. contained in the surface of a thing, as of a table, a floor, a picture, a field, &c. is found by multiplying the length into the breadth. If the length and breadth be equal, it is a square; then the measure of one of the sides, as of a room, is the root, of which the superficial content in the floor of the room is the second power." Hence to extract the square root of any number, is so to arrange the parts of that number that they may be in a square form.

Suppose you should have 144 square pieces of board, and you wished to know how many of those pieces would be on a side, if the whole were arranged into a square form. To determine this, you must extract the square root of 144 ; the first step of which is to point off the number into periods of two figures each. This shows how many figures the root will consist of, and is done on this principle, that the product of any two numbers can have, at most, but as many places of figures, as are in both the factors, and at least but one less.

$$\begin{array}{l} \dot{1}4\dot{4}(1 \\ 1 \\ \hline 44 \end{array}$$

Fig. 1.

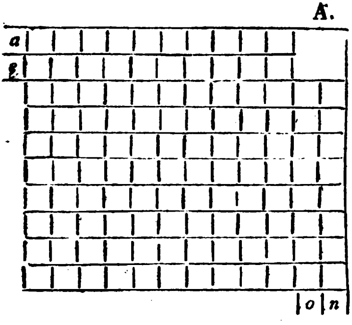


The left hand period being 1, the square of it will be 1, and likewise the root will be 1. But as we have nothing to do at present with the right hand period, we will omit it, and consider only the left hand period, which being in the place of hundreds, must be called 100 ; hence the operation, at present, will be to find the square root of 100. The root of 1, is 1, but as there are two periods in 100, there will be two figures in its root, and as the figure already obtained in the root is equal to its period, there is nothing remaining for the next period ; and as the next period consists wholly of ciphers, the next figure of the root will be a cipher, so that the root of 100 is 10. By this process we have disposed of 100 of the pieces into the form represented by Fig. 1, viz. 10 pieces on a side.

The reason for placing the square number underneath the period, and subtracting it from the period, as directed in the rule, is as follows. When we have obtained the root of the left hand period, we have disposed of as many pieces as the greatest square of left hand period represents, and by subtracting the square of the root from its period, we make it smaller by as many as the square of the root represents ; thus in the example given, 1 in the quotient represents 10, the square of which is 100, which *1, under* the left hand period, represents. This, subtracted *from* the left hand period, leaves 44 ; so that 100 pieces have been disposed of as represented by Fig. 1, and 44 pieces are now to be added to it, in such manner that the square form will be preserved. To do this, the rule directs to "*place the double of the root already found on the left hand of the dividend for a divisor.*"

$$\dot{1}4\dot{4}(12$$
$$\underline{1\phantom{44}}$$
$$22)44$$
$$\underline{\phantom{22)}44}$$

Fig. 2.



Now the first figure of the root shows the number of pieces there are on a side of Fig. 1, viz. 10. In order to preserve the square form the additions must be made on two adjoining sides of the square, as in Fig. 2. Now it is evident that if there were just 20 pieces left, after disposing of 100, there would be just enough to make a row on two sides of Fig. 1, and if there were 40 pieces left, they would make two rows, on two sides, as represented by the rows *a*, *e*, and *o*, *n*, Fig. 2 Hence the reason of placing the double of the root on the left of the dividend for a divisor. In making the additions *a*, *e*, and *o*, *n*, you will observe there is a deficiency, A. which is not filled. To fill this deficiency, the rule directs to "*except the right hand figure*," and likewise to "*place the quotient figure on the right hand of the divisor.*' Now the dificiency A. must be limited by the additions *a*, *e*, and *o*, *n*, consequently the figure, expressing the width of these additions, expresses the root of this deficiency, which, multiplied into itself, gives the superficial contents of the deficiency. Thus Fig. 2 shows the disposition of 144 pieces into a square form.

*Pup.* This appears much plainer than I at first imagined; there is nothing which is at all difficult.

*Tut.* After performing a few examples, we will proceed to examine the cube root.

What is the square root of 176248 ?

```
      ·  ·  ·
      176248(419,8 &c. Ans.
      16
     ----
    81)162
        81
      -----
   829)8148
       7461
      -----
  8388)68700
       67104
       -----
         596
```

What is the square root of 184,2 ?

```
      ·  ·  ·  ·
      184,2000(13,57
      1
     ---
   23)84
      69
     ----
  265)1520
      1325
     -----
 2707)19500
      18949
      -----
        551 remainder.
```

What is the square root of 10342656 ? Ans. 3216.

What is the square root of $\frac{5}{12}$ ? Ans. 0,645497.

# TO EXTRACT THE CUBE ROOT.

The extraction of the cube root, is the finding a number, which multiplied into its square, will produce the given number.

### *Rule.*

"Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

"Find the greatest cube in the left hand period, and put its root in the quotient.

"Subtract the cube thus found, from the said period, and to the remainder bring down the next period, and call this the dividend.

"Multiply the square of the quotient by 300, calling it the triple square, and the quotient by 30, calling it the triple quotient, and the sum of these call the divisor.

"Seek how often the divisor may be had in the dividend, and place the result in the quotient.

"Multiply the triple square by the last quotient figure, and write the product under the dividend; multiply the square of the last quotient figure by the triple quotient, and place this product under the last; under all, set the cube of the last quotient figure, and call their sum the subtrahend.

"Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before, and so on till the whole be finished."

The same rule will be observed for continuing the operation, and for extracting the roots of fractions, as in the square root.

What is the cube root of 10648?

```
                             .  .
 300     30            10648(22
   4      2            8
 ----    --           ----
1200     60       1260)2648
  60                   2400
 ----                   240        [tient figure.
1260                      8  cube of the last quo-
                       ----
                       2648

          1200 triple square.
             2
          ----
          2400
                    60 triple quotient.
                     4 square of the quotient.
                   ---
                   240
```

*Pup.* I understand the reason of obtaining the first figure of the root as we do, and of subtracting its cube from the left hand period; but I do not understand why the divisor is obtained as it is.

*Tut.* In order to give you a distinct understanding of the whole work, we will prepare blocks in the following manner, and build them up into a cubic form, as directed by the rule.

First make a cubic block of any given size, and mark it with the letter A. Then make three other blocks of a *square* form, of an indefinite thickness, but all equal to each other, each of which will just cover one side of the block A. and mark them, B. C. and D. Place these blocks on three adjoining sides of the block A. when there will be deficiencies at the three points where the blocks B. C. and D. meet. These deficiencies must be filled with three other blocks, each of which must be just equal in length to one side of the block A. and mark these blocks with E. F. and G. When the blocks E. F. G. are put in their places, there will be a deficiency at the place where the ends of these blocks meet. This deficiency must be filled with another block, which mark H. To illustrate the rule, I will take the number before used, and extract the cube root of it, explaining every part.

```
10648(2
 8
 —
 2
```

In the first place I seek the greatest cube in the left hand period, and place its root, 2, in the quotient. The cube of 2 is 8, which I place under the left hand period, and subtract it therefrom, which leaves a remainder of 2. Now as there are two periods in the given number, there must be 2 figures in the root, consequently, 2, in the quotient, does not express 2, merely, but 20; and the cube of 20 is 8000, which 8, under the period 10, represents; thus 8000 of the parts of 10648, are disposed of into a cubic body. the length of each side of which is equal to 20 of those parts, and to render the explanation more plain, we will consider these parts as cubic feet, so that each side *of this body* is 20 feet square, and this body we will have *represented* by the block A. Now as each side of this block is 20 feet square, there are 400 feet on each side of it. Now 8000 feet are disposed of in this block, consequently there are 2648 cubic feet to be added to the block A. in such a manner, that its cubic form will be preserved. To do this, the additions must be made to three sides of the block, and these additions are represented by the blocks, B. C. and D. each of which containing 400 feet, the sum of the whole is 1200. Thus it is evident, that if there were 1200 feet more, there would be just enough to cover three sides of the block A.; and it is to find the contents of these three sides, that the rule directs to "*multiply the square of the quotient by* 300." The square of the quotient shows the superficial contents of one side of the block A. viz. 400, for 2 in the quotient is in reality 20, and 20 × 20 = 400, and it is because the cipher is not annexed to the quotient figure, that we are directed to multiply the square of the quotient by 300 instead of 3, as in the following work.

| | | | |
|---|---|---|---|
| 4 | 10648(2 | 1200 | 60 |
| 300 | 8 | 2 | 4 |
| 1200 triple square. | 1260)2648 | 2400 | 240 |
| | 2400 | | |
| 2 | 240 | | |
| 30 | 8 cube of the root. | | |
| 60 triple quotient. | 2648 subtrahend. | | |
| 1200 | | | |
| 1260 divisor. | | | |

The rule directs to "*multiply the quotient by* 30." This is to obtain the contents of the blocks E. F. and G. and the sum of these is taken for a divisor, because the number of times the dividend contains the divisor, will be equal to the number of feet, the additions to the block, A. are in thickness.

The rule next directs to "*multiply the triple square by the last quotient figure.*" Now the triple square represents the superficial contents of the three blocks B. C. and D. and the last quotient figure shows the thickness of those blocks, consequently, multiplying the triple square by the last quotient figure gives the cubic contents of those blocks. Then the rule directs to "*multiply the square of the last quotient figure by the triple quotient.*" Now the triple quotient is the length of the blocks E. F. and G. and the quotient figure shows their breadth, and their thickness; hence multiplying the square of the last quotient figure by the triple quotient, gives the cubic contents of these blocks. Next we are directed to cube the last quotient figure. This cube shows the cubic contents of the block H. and the sum of these is equal to the cubic contents of the blocks B. C. and D.; and E. F. and G.

*Pup.* This appears much plainer than I expected it would, before I heard your explanation. I can see no difficulty in finding the cube root of any number, if the explanation is well attended to.

*Tut.* It is not so difficult to extract the cube root as it appears to be to those unacquainted with the reasons of the rule. If you understand the nature of the rule you will never be troubled in performing any operations in it.

~~What is the cube root of 1092727? Ans. 103.~~

---

*Questions.*

What is involution?

What is the power of a number?

What is the small figure, which denotes the order of the power, called?

How do you find the power of any number?

What is evolution, *or* the extraction of roots?

Why is the given number pointed into periods of two figures each, for extracting the square root?

Why do you subtract the square of the root from the period in which it was taken?

Why do you double the root for a divisor?

In dividing, why do you except the right hand figure of the dividend?

Why do you place the root in the quotient, and at the right of the divisor?

How is a number pointed, when accompanied by decimals?

*How is* the root of a vulgar fraction found?

What is it to extract the cube root?

When you have found the first figure of the root, why do you subtract its cube from the period in which it was taken?

Why do you multiply the square of the quotient by 300?

Why do you multiply the quotient by 30?

Why should the sum of the triple square and triple quotient be taken for a divisor?

Why do you multiply the triple square by the last quotient figure? the square of the last quotient figure by the triple quotient? and cube the last quotient figure, and take their sum for a subtrahend?

---

*Examples for Practice.*

1. What is the square root of 30138696025?
Ans. 173605.

2. What is the square root of 234,09? Ans. 15,3.

3. What is the square root of ,045369? Ans. ,213.

4. What is the square root of 964,5192360241?
Ans. 31,05671.

5. A general has an army of 4096 men; how many must he place in rank and file to form them into a square?
Ans. 64.

6. If the area of a triangle be 160, what is the side of a square equal in area thereto? Ans. 12,649†.

7. There is a circle whose diameter is 4 inches; what is the diameter of a circle 5 times as large?
Ans. 8,94 inches.

To solve questions of this kind, square the given diameter; then multiply this square by the given proportion, and extract the root of the product, which will be the answer. When the circle of the required diameter is to be less than the circle of the given diameter, the square of the given diameter must be divided by the proportion.

8. If the diameter of a circle be 20 feet, what is the diameter of a circle one third as large? Ans. 11,54 ft.

9. The eaves of a house are 18 feet high; there is a wharfing in front of the house 12 feet wide; what is the length of a ladder that will just reach from the edge of the wharfing to the eaves of the house? Ans. 21,63 ft.

In every right angled triangle, like that formed by the house, the wharfing, and the ladder, the square of the hypotenuse, or slanting side, is equal to the sum of the squares of the other two sides.

10. The height of a tree growing in the centre of a circular island, 44 feet in diameter, is 75 feet, and a line stretched from the top of it over to the hither edge of the water, is 256 feet; what is the breadth of the stream, provided the land on each side of the water be level?
Ans. 222,76 ft.

11. Two ships sail from the same port; one goes due north 45 leagues, and the other due west 76 leagues; how far are they asunder? Ans. 88,32 leagues.

12. What is the cube root of 16194277? Ans. 253.

13. What is the cube root of 27054036008?
Ans. 3002.

14. What is the cube root of ,0001357?
Ans. ,05138 &c.

15. What is the cube root of $\frac{2}{3}$? Ans. ,873 &c.

16. If a cube of silver, whose side is 4 inches, be worth $150; I demand the side of a cube of the like silver, which shall contain 3 times as much? Ans. 6,349 in.

17. If a globe of lead, 8 inches in diameter, weigh 3 cwt.; what is the weight of another globe 4 times as large? Ans. 24 cwt.

# CONVERSATION XII.

## BARTER AND ALLIGATION.

*Tut.* What is barter?

*Pup.* Barter is exchanging one commodity for another *in such* a manner that neither party shall sustain loss.

*Tut.* You are right. When the prices of two commodities are given; to know how much of one commodity must be given for a given quantity of the other commodity;—

Find the whole price of the given commodity, and divide it by the price of the other commodity, and the quotient will be the answer.

How much sugar, at $5, per cwt. must be given for 350 yards of linen, at 25 cents per yard?

Ans. 17 cwt. 2 qrs.

When the bartering price of one commodity is rated above its ready money price, to find the bartering price of the other;—

Say, as the ready money price of one commodity is to the ready money price of the other, so is the bartering price of one, to the bartering price of the other.

A. and B. barter; A. has 150 gallons of brandy at $1,375 per gallon, ready money, but in barter he will have $1.50; B. has linen at 44 cents per yard, ready money; how must B. sell his linen per yard in proportion to A's bartering price, and how many yards are equal to A's brandy?

Ans. barter price is 48 cents, and he must give A. 468 yds. 3 qrs.

There are other methods of bartering, but they are very simple, and you will easily understand them, from what I have taught you of the above methods.

# ALLIGATION.

What is alligation?

*Pup.* Alligation is the mixing of simples of different qualities.

*Tut.* Alligation is a method of mixing two or more simples of different qualities or prices, so as to make a middle quality or price.

Alligation is of two kinds, *medial* and *alternate*.

## ALLIGATION MEDIAL

Is a method of finding the rate of the compound, when the rates and quantities are given; for which we have the following rule.

**Multiply each quantity by its rate, and divide the sum of their products by the sum of the quantities, and the quotient will be the rate of the compound required.**

If I mix 20 bushels of wheat at $1, per bushel, 16 bushels of rye at 75 cents per bushel, 12 bushels of barley at 50 cents per bushel, and 8 bushels of oats at 40 cents per bushel; what is the value of one bushel of this mixture?

$$\begin{array}{rcrcr} 20 & \times & 100 & = & 20,00 \\ 16 & \times & 75 & = & 12,00 \\ 12 & \times & 50 & = & 6,00 \\ 8 & \times & 40 & = & 3,20 \\ \hline 56 & & & & \end{array}$$

$$\begin{array}{r} 56)41,20(735\frac{5}{7} \\ 392 \\ \hline 200 \\ 168 \\ \hline 320 \\ 280 \\ \hline 40 \end{array}$$

Ans. $0,735$\frac{5}{7}$

A goldsmith melted together 8oz. of gold, 22 carats* fine, 1lb. 8oz. of 21 carats fine, and 10oz. of 18 carats fine; what is the fineness of the composition?

Ans. $22\frac{8}{19}$ carats fine.

◆

It frequently happens, that we have a number of simples, with their rates given, and wish to know what quantity of each it will take to make a mixture of a given rate; so that this is the reverse of alligation medial, and is called

---

* A carat is a twenty fourth part; 22 carats fine, means $\frac{22}{24}$ of *pure* metal. A carat is also divided into four parts, called grains *of a carat.*

# ALLIGATION ALTERNATE.

If a man have corn worth $1, per bushel, and oats worth 50 cents per bushel, what quantity of each must he take, to make a mixture worth 75 cents per bushel?

Now if you take one bushel of corn, and mix it with one bushel of oats, there will be two bushels, the whole price of which will be $1,50, and one bushel will be 75 cents; and if even quantities of each be mixed, the price of the mixture will be 75 cents per bushel, whatever the quantity may be.

Again, if you have corn at 84 cents per bushel, rye at 72 cents, barley at 48 cents, and oats at 36 cents per bushel, and you wish to make a mixture which shall be worth 60 cents per bushel, you must so accommodate the mixture, that what is lost by diminishing the quantities, whose prices exceed the price of the mixture, may be gained by increasing those quantities whose prices are less than the price of the mixture; which may be done by linking the prices together in the following manner, and taking the difference between the price of each simple, and the price of the mixture, and writing it against the price with which this price is linked.

| | | |
|---|---|---|
| 60 | 36 | 24 bushels at 36 cts. |
| | 48 | 12 bushels at 48 cts. |
| | 72 | 12 bushels at 72 cts. |
| | 84 | 24 bushels at 84 cts. |

Proof by alligation medial.

36 × 24 = $8,64
48 × 12 = 5,76
72 × 12 = 8,64
84 × 24 = 20,16

72 ) 43,20(60
432

0

Hence the following rule;—

Write the prices of the simples in a column under each other, the least uppermost, and place the price of the compound at the left of the column. Connect or link with a continued line the price *of each simple*, which is less than that of the compound, with one

**or any number of those that are greater than the compound; and each greater price with one or any number of the less. Write the difference between the price of the mixture, and that of each of the simples, opposite to the prices with which they are respectively linked. Then if only one difference stand against any price, it will be the quantity belonging to that price; but if there be several, their sum will be the quantity.**

A goldsmith would mix gold of 19 carats fine with some of 16, 18, 23 and 24 carats fine, so that the compound may be 21 carats fine; what quantity of each must he take? Ans. 5 oz of 16 carats fine, 5 oz. of 18, 5 oz. of 19, 10 oz. of 23, and 10 oz. of 24 carats fine.

How much barley at 40 cents, rye at 60 cents, and wheat at 80 cents per bushel, must be mixed together, that the compound may be worth 62½ cents per bushel?

Ans. { 17½ barley,
17½ rye,
25 wheat.

---

When the whole composition is limited to a certain quantity, find an answer by linking as before; then say, as the sum of the quantities, thus found, is to the given quantity, so is each quantity, found by linking, to the required quantity of each.

Suppose you have corn at 75 cents, and oats at 40 cents per bushel; how many bushels of each must be taken to make 120 bushels worth 60 cents per bushel?

60 { 50 } 15   25 : 120 :: 10 : 48 bushels of corn.
   { 75 } 10   25 : 120 :: 10 : 72 bushels of oats.
          25

A goldsmith has gold of 14, 16, 18, and 22 carats fine, and wishes to melt together, of all these sorts, so much as will make a mass of 50 oz. 20 carats fine; how much must he take of each sort? Ans. 5$\frac{5}{9}$ oz. 14 carats fine; 5$\frac{5}{9}$ 16; 5$\frac{5}{9}$ oz. 18; and 33⅓ oz. 22 carats fine.

---

When one of the ingredients is limited to a certain quantity;

**Find the difference between each price and the price of the *mixture* as before; then say, as the difference of that simple,**

whose quantity is given, is to the rest of the differences severally, so is the quantity given to the several quantities required.

How much gold of 16, 20, and 24 carats fine, and how much alloy, must be mixed with 10 oz. of 18 carats fine, that the composition may be 22 carats fine?

$$\begin{array}{rl|l} \text{Alloy,} & 00 & 2 \\ & 16 & 2 \\ 22 & 18 & 2 \\ & 20 & 2 \\ & 24 & 2 + 4 + 6 + 22 = 34. \end{array}$$

2 : 2 :: 10 : 10 oz. alloy.
2 : 2 :: 10 : 10 oz. 16 carats fine.
2 : 2 :: 10 : 10 oz. 20 carats fine.
2 : 34 :: 10 : 170 oz. 24 carats fine.

## *Questions.*

How is barter useful?

When the prices of two commodities are given, how do you find how much of one must be given for the other?

When the bartering price of one commodity is rated above its ready money price, how do you find the bartering price of the other?

What is alligation?

How many kinds of alligation are there?

What is alligation medial?

What is the rule for it?

What is alligation alternate?

When you have several commodities, with their rates or prices given, how do you find what quantity of each must be taken to make a mixture of a given rate?

When the whole composition is limited to a certain quantity, how do you find the required quantity of each commodity?

When one of the ingredients is limited to a certain quantity, how do you find the required quantity of each commodity?

## *Examples for Practice.*

1. A. has 200 yards of linen at 25 cents per yard, ready money, which he barters with B. at 30 cents per yard, taking buttons at 11 cents per gross, which are worth but 8 cents; how many gross of buttons will pay for the linen; who gets the best bargain; and by how much? Ans. 545,45 gross, B. gains $6,364.

2. How much corn at $1 per bushel, must I give in exchange for 8 cwt. of sugar, at $9,45 per cwt.?
Ans. 75 bu 2 pe. $3\frac{1}{5}$ qts.

3. How much gold of 15, 17, and 22 carats fine, must be mixed with 5 oz. of 18 carats fine, so that the composition may be 20 carats fine?
Ans. 5 oz. of 15 carats fine, 5 of 17, and 25 of 22.

4. A. merchant would mix 4 sorts of wine, of several prices, viz. at 75 cents, $1.25, $1,50, and $1,625 per. gallon; of these, he would have a mixture of 60 gallons, worth $1,167 per gallon; what quantity of each sort must he take?
Ans. $24\frac{1440}{2125}$ gal. at 75 cts. $11\frac{1645}{2125}$ gal. of each of the others.

◆

# GAUGING.

I will now give a few practical rules for Gauging, and the Mechanical Powers, with several interesting Mathematical Problems.

I have omitted the demonstrations to Gauging, as it would require a more extensive knowledge of mathematics to understand them, than the pupil can, at present, be supposed to possess, or the limits of this work will allow me to give. The practical part is so easy, that the pupil will have no difficulty in obtaining a practical knowledge of the rule without the demonstrations.

Gauging, is taking the dimensions of a cask in inches, to find its contents in gallons, by the following

### *Method.*

To the head diameter add two thirds of the difference *between* the head and bung diameters, and the sum will be the mean diameter; but if the staves be but little curving, add only six tenths of this difference. Square the mean diameter, which, multiplied by the length of the cask, and the product divided by 294, for wine, or by 359 for ale, the quotient will be the answer in gallons.

Note. There is a difference in the thickness of the heads of different casks, for which proper allowances must be made in taking the length The head diameter must be taken close to the chimes; and for small casks, add 3 tenths of an inch; for casks of 40 or 50 gallons, 4 tenths; and for larger casks, 5 or 6 tenths; and the sum will be very nearly the head diameter within.

*Examples.*

How many ale or beer gallons will a cask hold, whose bung diameter is 28 inches, head diameter 22 inches, and whose length is 38 inches?

```
28 bung diam.          22 head diam.
22 head diam.           4 two thirds diff.
——                     ——
 6                     26
                       26
                      ———
                      156
                      52
                      ———
                      676
                       38
                      ———
                     5408
                    2028
                    ————
              359)25688(71 199/359 gallons.
```

How many wine gallons will a cask hold, whose bung diameter is 20 inches, head diameter 17 inches, and whose length is 24 inches? Ans. 29 gal. $1\frac{258}{294}$ qts.

## MECHANICAL POWERS OF THE LEVER.

To find what weight may be raised or balanced by any given power, say; As the distance between the body to be raised or balanced, and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied, so is the power to the weight which it will balance.

If a man weighing 160 lb. rest on the end of a lever, 12 feet long, what weight will he balance on the other end, if the prop be 1 foot from the weight.

1 : 11 : : 160 : 1760 lb. Ans.

At what distance from a weight of 1760 lb. must a prop be placed, so that a weight of 160 lb. applied, 11 ft. from the prop, will raise it? 1760 : 160 : : 11 : 1 ft. Ans.

In giving directions for making a chaise, the length of the shafts between the axletree and back board, being settled at 9 feet, a dispute arose whereabout on the shafts the centre of the body should be fixed. The chaise-maker advised to place it 30 inches before the axletree; others supposed 20 inches would be a sufficient encumbrance for the horse: Now supposing two passengers to weigh 3 cwt. and the body of the chaise $\frac{3}{4}$ cwt. more; what will the beast, in both these cases, bear more than his harness?

Weight of the chaise and passengers $3\frac{3}{4}$ cwt.=420 lbs. and 9 feet=108 inches.

$$\text{Then, as } \overset{\text{in.}}{108} : \left\{\begin{matrix} 30 :: \\ 20 :: \end{matrix}\right\} 420 \left\{\begin{matrix} : \overset{\text{lb.}}{116\frac{2}{3}} \\ : 77\frac{7}{9} \end{matrix}\right\} \text{Ans.}$$

◆

## OF THE WHEEL AND AXLE.

As the weight applied to the axle is to the weight or power applied to the wheel, so is the diameter of the axle to the diameter of the wheel;

OR,

As the diameter of the wheel is to the diameter of the axle, so is the weight applied to the axle to the weight applied to the wheel.

If the diameter of the axle be 4 inches, and the weight applied to it be 20 lb. what must be the diameter of the wheel, so that 2 lb. at the wheel will just balance the 20 lb. at the axle?

$$\overset{\text{lb.}}{2} : \overset{\text{lb.}}{20} :: \overset{\text{in.}}{4} : 40 \text{ Ans.}$$

If the diameter of the axle be 6 inches, and that of the wheel be 48 inches, what weight applied to the wheel will balance 1268 lb. at the axle?

in. in. lbs.
48 : 6 :: 1268 : $366\frac{5}{6}$ lbs. Ans.

If the diameter of the wheel be 50 inches, and that of the axle 5 inches, what weight applied to the axle will balance 2 lb. at the wheel?

in. in. lb. lb.
5 : 50 :: 2 : 20 Ans.

## OF THE SCREW.

The power is to the weight to be raised, as the distance between two threads of the screw is to the circumference of a circle described by the power applied at the end of the lever.

The diameter of a circle is to its circumference as 7 is to 22, or more correctly, as 113 is to 355: Hence to find the circumference of the circle, described by the power at the end of the lever, say; As 7 is to double the lever, so is 22 to the circumference; or as 113 is to double the lever, so is 355 to the circumference; then as that circumference is to the distance between the spiral threads of the screw, so is the weight to be raised, to the power which will raise it, abating $\frac{1}{3}$ of the effect of the machine for friction.

There is a screw whose threads are one inch asunder; the lever by which it is turned, 72 inches long, and the weight to be raised, 3560 lb; what power must be applied to the end of the lever, sufficient to turn the screw, that is, to raise the weight?

7 : 144 :: 22 : $452\frac{4}{7}$ circumference.

in. in. lb.
Then, as $452\frac{4}{7}$ : 1 :: 3560 : $7\frac{2744}{3168}$ lb. Ans.

If the lever be 72 inches, the circumference $452\frac{4}{7}$ inches, the threads 1 inch asunder, and the power $7\frac{2744}{3168}$ lb.= 7,865lb; what is the weight to be raised?

in. in. lb.
1 : 452,571 :: 7, 865 : 3559,360915 Ans.

There is a loss of a few ounces, by the fractions.

---

## PROBLEMS.

The diameter of a circle being given, to find the circumference.

As 7 is to the diameter, so is 22 to the required circumference; or as 113 is to the diameter, so is 355 to the required circumference.

There is a circle whose diameter is 12 feet, what is its circumference?

7 : 12 :: 22 : 37,7142.

or, 113 : 12 :: 355 : 37,6991.

The circumference of a circle being given, to find the diameter.

As 22 is to the circumference, so is 7 to the required diameter; or as 355 is to the circumference, so is 113 to the required diameter.

There is a circle whose circumference is 48 feet; what is its diameter?

22 : 48 :: 7 : 15,272

OR,

355 : 48 :: 113 : 15,278.

To find the area of a circle;—

Multiply half the diameter by half the circumference, and the product will be the area.

There is a circle whose diameter is 15,278 feet; and its circumference 48 feet; what is its area?

```
 7,639 half the diameter.
    24 half the circumference.
 -----
  3056
 15278
 -----
183,336 feet = area.
```

**To find the solid contents of a sphere or globe ;—**

Multiply the circumference by the diameter, which will give the superficial content ; then multiply the surface by *one sixth* of the diameter, and the product will be the solidity.

The diameter of a globe is 8 feet ; what is its solid content ?

8 : 7 :: 22 : 25,142 × 8 = 201,136 × 1,333 = 268,114288 ft. Ans.

**To find how much a round tree, that is equally thick from end to end, will hew to, when made square ;—**

Extract the square root of twice the square of the semi-diameter, which root will be the side of the stick when made square.

There is a round stick of timber 24 inches in diameter ; how large square will it hew ?

```
       12
       12
      ----
      144
        2
     -----     in.
      288(16,97 the side of the stick.
      1
     ------
   26)188
      156
     ------
  329)3200
      2961
     -------
 3387)23900
      23709
      -----
        191
```

**To measure the solidity of any irregular body whose dimensions cannot be taken ;—**

Put the body to be measured into some regular vessel, and fill it with water ; then take out the body, measure the fall of water in the vessel, which will be the solidity of the body.

## *Miscellaneous Questions.*

1. What number added to the thirty first part of 3813, will make the sum 200? Ans. 77.

2. What difference is there between twice 5 and twenty, and twice twenty five? Ans. 20.

3. The remainder of a division is 325, the quotient 467, and the divisor is 43 more than the sum of both; what is the dividend? Ans. 390270.

4. What will 14 cwt. beef cost at 5 cents per pound? Ans. $78,48.

5. In an orchard of fruit trees $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 60 of them peaches, and 40 cherries; how many trees does the orchard contain? Ans. 1200.

6. What length must be cut off a board $8\frac{3}{8}$ inches broad, to contain a square foot? Ans. $17\frac{13}{67}$ inch.

7. A father divided his estate in the following manner; to A. he gave 4 so often as B. had 6, and to C. 7 so often as 9 to D; what was the sum divided if A's share was $600? Ans. $ 2600.

8. A father dying left his son a fortune, $\frac{1}{4}$ of which he ran through in 8 months; $\frac{3}{7}$ of the remainder lasted him 12 months longer, when he had $900 left; what did his father bequeath him? Ans. $2100.

9. A merchant begins the world with $500, and finds that by his distillery he clears $5000, in 6 years; by his navigation, $5000 in $7\frac{1}{2}$ years; and that he spends in gaming $5000 in 3 years; how long will his estate last? Ans. 30 years.

10. A. can do a piece of work alone in 7 days, and B. in 12; in what time will both, working together, finish it? Ans. $4\frac{8}{19}$ days.

11. A. and B. are on opposite sides of a circular field 268 poles about; they begin to go round it, both the same way at the same instant of time; A. goes 22 rods in 2 minutes, and B. 34 rods in 3 minutes; how many times will they go round the field before the swifter overtakes the slower? Ans. A $16\frac{1}{2}$ times.
B. 17 times.

12. A. and B. together can do a piece of work in 20 days; with the assistance of C they can do it in 12 days, in what time would C. do it by himself?

Ans. 30 days.

13. Suppose a cistern has a pipe that conveys 4 gallons, 2 qts. into it in an hour, and another that lets out 2 gallons 1qt. 1pt in an hour; if the cistern contain 84 gallons, in what time will it be filled?

Ans. 39 h. 31 m. $45\frac{15}{17}$ sec.

14. If a man spend $22,8125 in one year, what does he spend per day? Ans. $0,0625.

15. A guardian paid $4800, for $3692,309 which he had in his hands 6 years: What rate of interest did he pay?

Ans. 5 per cent.

16. What number is that from which if you take $\frac{2}{7}$ of $\frac{3}{8}$, and to the remainder add $\frac{7}{16}$ of $\frac{1}{20}$, the number will be 10? Ans. $10\frac{191}{2240}$.

17. A water tub holds 73 gallons; the pipe, which conveys the water to it usually admits 7 gallons in 5 minutes; and the tap discharges 20 gallons in 17 minutes. Now, supposing both these to be carelessly left open, and the water to be turned on at 4 o'clock in the morning; a servant, at 6, finding the water running, puts in the tap; in what time, after this accident, will the tub be filled? Ans. 32 min. $58\frac{118}{119}$ sec.

18. A hare starts 12 rods before a hound; but is not perceived by him till she has been up 45 seconds; she scuds away at the rate of 10 miles an hour; and the dog, on view, makes after her at the rate of 16 miles an hour; how long will the course hold, and what space will be run over, from the spot where the dog started?

Ans. 2288 ft. $97\frac{1}{2}$ seconds.

19. Required the number, from which, if 7 be subtracted, and the remainder be divided by 8, and the quotient be multiplied by 5, and 4 added to the product, the square root of the sum extracted, and three fourths of that root cubed, the cube, divided by 9, the last quotient, may be 24? Ans. 103.

20. Two men depart from the same place; the one goes East, and the other North; one goes 20 miles per

day, and the other 25 miles ; how far apart are they on the 4th day after their departure ? Ans. 128,062 miles.

21. Suppose a lighthouse built on the top of a rock ; the distance between the place of observation and that part of the rock level with the eye, 620 yards ; the distance from the top of the rock to the place of observation, 846 yards ; and from the top of the light house 900 yards ; the height of the light house is required ? Ans. 76,77 yds.

22. Sound, uninterrupted, moves at the rate of 1142 feet per second ; if the time between the lightning and thunder be one minute, at what distance was the explosion ? Ans. 12,977 &c. miles.

23. If the earth be 7911 miles in diameter, and the moon 2180 miles ; how many moons will it take to make one earth ? Ans. 47,788.

24. A. sets out for London precisely at the time when B. at Lincoln, sets forward for London, distant 100 miles ; after 7 hours they met on the road, and it then appeared, that A. had ridden $1\frac{1}{2}$ mile more than B At what rate an hour did each of them travel ? Ans. A. $7\frac{25}{28}$, B. $6\frac{11}{28}$.

25. There is an island 73 miles in circumference, and 3 footmen all start together to travel the same way around it ; A. goes 5 miles a day, B. 8, and C. 10 ; when will they all come together again ? Ans. 73. days.

26. A. B. and C. are to share $48000 in the proportion of $\frac{1}{3}$, $\frac{1}{4}$. and $\frac{1}{5}$. respectively ; but C's part being lost by his death, it is required to divide the whole sum properly between them ? Ans. A. $27428,571$\frac{3}{7}$. B. 20571,428$\frac{4}{7}$.

27. A man dying left his wife in expectation that a child would be afterwards added to the surviving family ; and making his will, ordered, that. if the child were a son, $\frac{2}{3}$ of his estate should belong to him, and the remainder to his mother ; but if it were a daughter, he appointed the mother $\frac{2}{3}$, and the child the remainder. But it happened, that the addition was both a son and a daughter, by which the widow lost in equity, $2400 more than if there had been only a girl. What would have been her dowry had she had only a son ? Ans. $2100.

28. A man died leaving an estate of $1800, which he divided between his three sons in the following manner. They were of 12, 16, and 18 years of age, and were to

receive such sums at their father's decease, as put at interest, would give them equal sums, when they arrived at 21 years of age. What did each one receive?

Ans. $673,142 &c.
611,028 &c.
515,803 &c.

29. When first the marriage not was tied
Betwixt my wife and me,
My age with hers did so agree,
As nineteen does with eight and three;
But after ten and half ten years,
We man and wife had been,
Her age came up so near to mine,
As two times three to nine.

What were our ages at marriage?
Ans. 57 and 33.

### AMUSING AND INSTRUCTIVE QUESTIONS.

A man bought a piece of ground 30 feet long and 20 feet wide, on which he set a barn and house, each 30 feet long and 20 feet wide; how did he set them?

Said Harry to Edmund, I can place four 1's so that, when added, they shall make precisely 12; can you do so too?

Says Edmund to Harry, the upper half of a carriage wheel, moves faster than the under half; can you tell me the reason?

Charles said, that his instructor asked him if it would take any more boards to build a fence over the top of a hill, than it would to build one of the same height, (by a plumb line) across the base of the same hill: he told his instructer that it would take no more boards; and also that no more trees could grow on a hill, than could grow on its base, or on a piece of ground equal to its base, although the surface of the hill should be double the surface of its base. His instructer told him that he was correct.

If a cannon be raised one mile high, with the mouth pointing directly towards the earth, and it be discharged in that position, where will the ball move the swiftest, *through the first or last half mile from the cannon's mouth?*

Three travellers met at a caravansary, or inn, in Persia; and two of them brought their provisions along with them, according to the custom of the country; but the third, not having provided any, proposed to the others that they should eat together, and he would pay the value of his proportion. This being agreed to, A produces 5 loaves, and B. 3 loaves, which the travellers eat together, and C. paid 8 pieces of money, as the value of his share, with which the others were satisfied, but quarrelled about the dividing of it. Upon this the affair was referred to the Judge, who decided the dispute by an impartial sentence. Required his decision?

Come, sprightly youth, and with me rove
In yonder lawn to plant a grove;
Its curious form lay out with care,
Just as my fancy pictures here;
This sylvan spot you must compose
Of nineteen trees in nine straight rows;
And to adorn the shaded land,
In each row five trees must stand.
Your task performed, the central tree
We'll consecrate to LIBERTY.

# FORMS OF NOTES, BONDS, RECEIPTS, ORDERS, and DEEDS.

## OF NOTES.

### No. 1.

*Tatta*, *Sept. 4th*, 1823. For value received, I promise to pay to John Pilfort, or order, forty dollars on demand, with interest.

Attest, **CHARLES HAMMER.**

**HEMAN GEORGE.**

### No. 2.

*Astracan*, *August* 20th, 1823. For value received, I promise to pay to A. B. or order, one hundred dollars, on demand, with interest after four months. J. L.

### No. 3.

*Tatta*, *October* 4th, 1823. For value received, we jointly and *severally* promise to pay M. N. or bearer, one thousand dollars on *demand*, with interest.

**JOHN MAC.**

**HENRY BLOOM.**

No. 4.

*Port Jackson*, *Sept.* 6, 1823. For value received, I, C. D. of the town of ———, county of ———, and state of ———, promise to pay to F. H. of———, county of ———, and state of———, three hundred dollars on demand, with interest. C. D.

## *Remarks.*

When a note is written to pay him, or order, as in No. 1. and 2. the person to whom the note is given, may endorse it, that is, write his name on the back of it, and sell it to whom he pleases; then whoever buys the note, may call on the signer of the note, and if he is unable to pay, or declines doing it, the endorser must pay it.

When a note is written to pay him or bearer as in No. 3, any person holding the note may sue, and recover it of the signer. But if the note is not written, to pay him or order, it is not negociable, that is, it cannot be sold or transferred from one person to another; and if it is not written, to pay him or order, or him or bearer, none but the person to whom the note is given, can sue and recover the same.

When a note is written, on demand, it is recoverable, any time at the option of the creditor; but if it be written, payable after the expiration of a certain time, as in No. 2, it cannot be recovered till the expiration of that time.

It is unnecessary, in writing notes, to mention the rate of interest, unless it be below the rate established by law; for no greater rate than legal interest is recoverable in law, and when no rate is mentioned, legal interest is understood.

When a note is written, payable at a certain time, interest does not commence on it till the expiration of that time, unless particularly mentioned in the note.

When a note is given jointly and severally by two or more persons, or by a company, the whole note may be recovered of any individual of the company or signers.

When a note is given, payable at a certain time, in any specified article, as corn, rye, &c. if the article is not offered before the time expires, the creditor is not obliged to receive the article in payment, but may recover the amount in money.

## OF BONDS.

### *A Bond with a condition from one to another.*

KNOW all men by these presents, That I, A. B. of ———, in the county of ———, am held firmly bound to C. D. of H. in the sum of two hundred dollars, to be paid to the said C. D. his attorney, his executors, administrators or assigns, to which payment, well and truly to be made, I bind myself, my heirs, executors and administrators, firmly by these presents. Sealed with my seal. Dated the fourth day of ———, in the year of our Lord one thousand, eight hundred and twenty-three.

*The condition* of this obligation is such, that if the above-bound A. B. his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid unto the above-named C. D. his executors, administrators, or assigns, the full sum of two hundred dollars, with legal interest for the same, on or before the fourth day of —— next ensuing the date hereof; then this obligation to be void, or otherwise to remain in full force and virtue.

### *A Condition of a Counter Bond, or Bond of Indemnity, where one man becomes bound for another.*

THE condition of this obligation is such, That whereas the above-named A. B. at the special instance and request, and for the only proper debt of the above bound E. F. together with the said E. F. is, and by one bond or obligation, bearing equal date with the obligation above written, held and firmly bound unto M. N. of H. in the penal sum of —— dollars, conditioned for the payment of the sum of, &c. with legal interest for the same, on the —— day of —— next ensuing the date of the said in part recited obligation, as in and by the said in part recited bond, with the condition thereunder written, may more fully appear: If therefore, the said E. F. his heirs, executors, or administrators, do and shall well and truly pay, or cause to be paid unto the said M. N. his executors, administrators, or assigns, the said sum of, &c. with legal interest of the same, on the said —— day of, &c. next ensuing the date of the said in part recited obligation, according to the true intent and meaning, and in full discharge and satisfaction of the said in part recited bond or obligation: Then this obligation to be void, or otherwise to remain in full force and virtue.

## OF RECEIPTS.

### No. 1.

*Antonia, Sept.* 12, 1823. Received of Mr. Theodore Allswell, twenty dollars in full of all accounts. **Abel Zealous.**

### No. 2.

*Anton*[illegible], *Sept.* 12, 1823. Received of Mr. Abel Zealous, ten dollars in full of all demands. **Theodore Allswell.**

### *Receipt for an Endorsement on a Note.*

*Mexico, Sept.* 12, 1823. Received of Mr. S. G. (by the hand of M. L.) thirty dollars, which is endorsed on his note of March 14, 1814. L. T.

### *Receipt for Interest due on a Bond.*

*Obolsk, Sept.* 12, 1823. Received this day of—— of Mr. I. L. the sum of twelve dollars in full of one year's interest of two hundred dollars, due to me on the —— day of —— last, on bond from the said I. L. I say received by me, H. G.

*Receipt for money received on account.*

*Obolsk, Sept.* 12, 1823. Received of Mr. O. H. forty dollars on account. O. M.

*Remarks.*

When two persons pass receipts, as in No. 1. and 2. a particular sum must be expressed in each receipt, but it is not necessary that it should be the exact sum received.

Receipts given in full of all accounts, cut off accounts only; but receipts given in full of *all demands*, cut off not only accounts, but obligations of all kinds.

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## OF ORDERS.

*Philadelphia, June* 4, 1823.

Mr. O. N. Sir, For value received, pay to G. M. ten dollars, and place the same to my account. L. F.

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*Allenton, August* 3, 1823.

Mr. I. R. For value received, pay to M. D. five dollars, and this, with his receipt, shall be your discharge from me. T. S.

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*Allenton, August* 6, 1823.

Messrs. J. & C. Please to send by the bearer, Mr. F. L. two hundred dollars, and place the same to my accompt.

G. M.

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## OF DEEDS.

### *A Warranty Deed.*

KNOW ALL MEN BY THESE PRESENTS, That I, A. B. of———, in the County of———, and Commonwealth of———, gentleman, for and in consideration of—— dollars, —— cents, paid to me by C. D. of———, in the county of———, and Commonwealth of———, yeoman, the receipt whereof I do hereby acknowledge, do hereby give, grant, sell and convey to the said C. D. his heirs and assigns, a certain tract and parcel of land, bounded as follows, viz.

[*Here insert the bounds, together with all the privileges and appurtenances thereunto belonging.*]

To have and to hold the same unto the said C. D. his heirs and assigns, to his and their use and behoof forever. And I do covenant with the said C. D. his heirs and assigns, that I am lawfully seized in fee of the premises, that they are free of all incumbrances, and that I will warrant and will defend the same to the said C. D. his heirs and assigns forever, against the lawful claims and demands of all persons.

In witness whereof, I hereunto set my hand and seal this——— of———, in the year of our Lord one thousand eight hundred and———

*Signed, sealed and delivered*
*in presence of*
E. F. G. H.

A.—— B.——

## *A Quitclaim Deed.*

KNOW ALL MEN BY THESE PRESENTS, That I, L. M. of, &c. in consideration of the sum of ——, to be paid by P. R. of, &c. the receipt whereof I do hereby acknowledge, have remissed, released, and forever quitclaimed, and do by these presents remit, release, and forever quitclaim unto the said P. R. his heirs and assigns forever. (*Here insert the premises.*) To have and to hold the same, together with all the privileges and appurtenances thereunto belonging, to him the said P. R. his heirs and assigns forever.

*In witness, &c.*

## *A Mortgage Deed.*

KNOW ALL MEN BY THESE PRESENTS, That I, W. B. of ——, in the County of ——, in the State of ——, Carpenter, in consideration of —— Dollars —— Cents, paid by R. S. of ——, in the County of ——, in the State of ——, Tailor, the receipt whereof I do hereby acknowledge, do hereby give, grant, sell and convey unto the said R. S. his heirs and assigns, a certain tract and parcel of land, bounded as follows, viz.

[*Here insert the bounds, together with all the privileges and appurtenances thereunto belonging.*]

To have and to hold the afore granted premises to the said R. S. his heirs and assigns, to his and their use and behoof forever. And I do covenant with the said R. S. his heirs and assigns, That I am lawfully seized in fee of the afore granted premises. That they are free of all incumbrances: That I have good right to sell and convey the same to the said R. S. And that I will warrant and defend the same premises to the said R. S. his heirs and assigns forever, against the lawful claims and demands of all persons. *Provided nevertheless*, That if I the said W. B. my heirs, executors, or administrators shall well and truly pay to the said R. S. his heirs, executors, administrators or assigns, the full and just sum of —— dollars —— cents, on or before the —— day of ——, which will be in the year of our Lord eighteen hundred and ——, with lawful interest for the same until paid, then this deed, as also a certain bond [*or note, as the case may be*] bearing even date with these presents given by me to the said R. S. conditioned to pay the same sum and interest at the time aforesaid, shall be void, otherwise to remain in full force and virtue In witness whereof, I the said, W. B. and E. B. my wife, in testimony that she relinquishes all her right to dower or alimony in and to the above described premises, hereunto set our hands and seals this —— day of ——, in the year of our Lord one thousand eight hundred and ——.

*Signed, sealed and delivered*
*in presence of*
H. J. L. P.

W.—— B——
E.—— B.——

THE END.